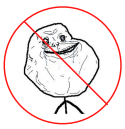
## Higher-Order Functions

## Announcements

## Office Hours: You Should Go!

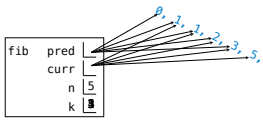
**You are not alone!**

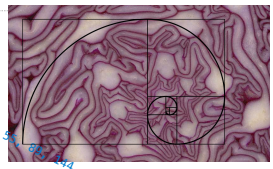


http://cs61a.org/office-hours.html

## Iteration Example

## The Fibonacci Sequence

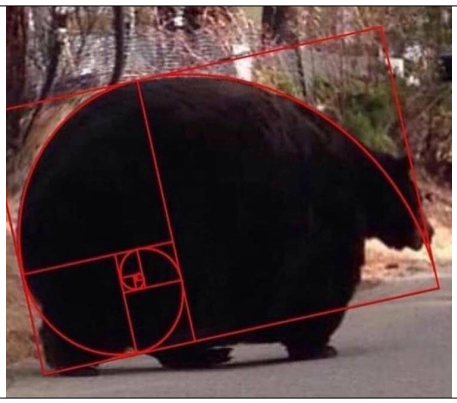


```
def fib(n):
    """Compute the nth Fibonacci number, for N >= 1."""
    pred, curr = 0, 1   # 0th and 1st Fibonacci numbers
    k = 1               # curr is the kth Fibonacci number
    while k < n:
        pred, curr = curr, pred + curr
        k = k + 1
    return curr
```

The next Fibonacci number is the sum of the current one and its predecessor

## Go Bears!



## Designing Functions

## Describing Functions

A function's *domain* is the set of all inputs it might possibly take as arguments.

A function's *range* is the set of output values it might possibly return.

A pure function's *behavior* is the relationship it creates between input and output.

```
def square(x):
    """Return X * X."""
```

*x is a number*

*square returns a non-negative real number*

*square returns the square of x*

## A Guide to Designing Function

Give each function exactly one job, but make it apply to many related situations

```
>>> round(1.23)      >>> round(1.23, 1)      >>> round(1.23, 0)      >>> round(1.23, 5)
1                    1.2                    1                      1.23
```

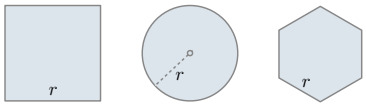
Don't repeat yourself (DRY): Implement a process just once, but execute it many times

(Demo)

## Generalization

## Generalizing Patterns with Arguments

Regular geometric shapes relate length and area.

**Shape:** 


**Area:** $1 \cdot r^2$ $\pi \cdot r^2$ $\frac{3\sqrt{3}}{2} \cdot r^2$

Finding common structure allows for shared implementation

(Demo)

## Higher-Order Functions

## Generalizing Over Computational Processes

The common structure among functions may be a computational process, rather than a number.

$$\sum_{k=1}^{5} k = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum_{k=1}^{5} k^3 = 1^3 + 2^3 + 3^3 + 4^3 + 5^3 = 225$$

$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} = 3.04$$

(Demo)

## Summation Example

```
def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
    225
    """
    total, k = 0, 1
    while k <= n:
        total, k = total + term(k), k + 1
    return total
```

Function of a single argument (*not called "term"*)

A formal parameter that will be bound to a function

The cube function is passed as an argument value

0 + 1 + 8 + 27 + 64 + 125

The function bound to term gets called here

## Functions as Return Values

(Demo)

## Locally Defined Functions

Functions defined within other function bodies are bound to names in a local frame

A function that returns a function

```
def make_adder(n):
    """Return a function that takes one argument k and returns k + n.
    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

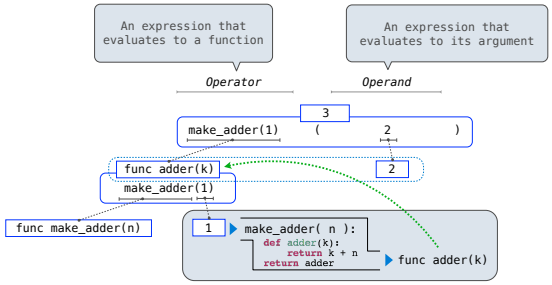
The name add_three is bound to a function

A def statement within another def statement

Can refer to names in the enclosing function



## Call Expressions as Operator Expressions






## Lambda Expressions

(Demo)

## Lambda Expressions

```
>>> x = 10
>>> square = x * x
>>> square = lambda x: x * x
>>> square(4)
16
```

An expression: this one evaluates to a number

Also an expression: evaluates to a function

Important: No "return" keyword!

A function with formal parameter x that returns the value of "x * x"

Must be a single expression

Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!



## Lambda Expressions Versus Def Statements

```
square = lambda x: x * x
```

**VS**

```
def square(x):
    return x * x
```

- Both create a function with the same domain, range, and behavior.
- Both bind that function to the name square.
- Only the def statement gives the function an intrinsic name, which shows up in environment diagrams but doesn't affect execution (unless the function is printed).




## Return

## Return Statements

A return statement completes the evaluation of a call expression and provides its value:

f(x) for user-defined function f: switch to a new environment; execute f's body

`return` statement within f: switch back to the previous environment; f(x) now has a value

Only one return statement is ever executed while executing the body of a function

```
def end(n, d):
    """Print the final digits of N in reverse order until D is found.

    >>> end(34567, 5)
    7
    6
    5
    """
    while n > 0:
        last, n = n % 10, n // 10
        print(last)
        if d == last:
            return None
```

(Demo)

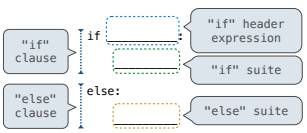


## Control

## If Statements and Call Expressions

Let's try to write a function that does the same thing as an if statement.

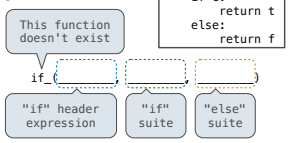
```
def if_(c, t, f):
    if c:
        return t
    else:
        return f
```




**Execution Rule for Conditional Statements:**

Each clause is considered in order.

1. Evaluate the header's expression (if present).
2. If it is a true value (or an else header), execute the suite & skip the remaining clauses.




**Evaluation Rule for Call Expressions:**

1. Evaluate the operator and then the operand subexpressions
2. Apply the function that is the value of the operator to the arguments that are the values of the operands

(Demo)



## Control Expressions

## Logical Operators

To evaluate the expression **<left> and <right>**:

1. Evaluate the subexpression **<left>**.
2. If the result is a false value **v**, then the expression evaluates to **v**.
3. Otherwise, the expression evaluates to the value of the subexpression **<right>**.

To evaluate the expression **<left> or <right>**:

1. Evaluate the subexpression **<left>**.
2. If the result is a true value **v**, then the expression evaluates to **v**.
3. Otherwise, the expression evaluates to the value of the subexpression **<right>**.

(Demo)



## Conditional Expressions

A conditional expression has the form

**<consequent> if <predicate> else <alternative>**

**Evaluation rule:**

1. Evaluate the **<predicate>** expression.
2. If it's a true value, the value of the whole expression is the value of the **<consequent>**.
3. Otherwise, the value of the whole expression is the value of the **<alternative>**.

```
>>> x = 0
>>> abs(1/x if x != 0 else 0)
0
```