Higher-Order Functions

Announcements

Office Hours: You Should Go!

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You are not alone!

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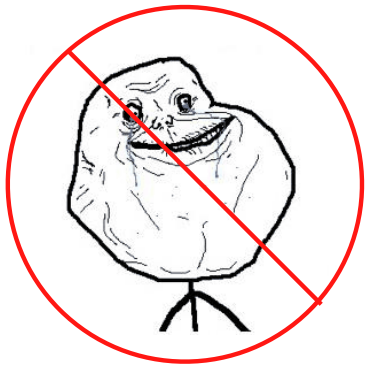

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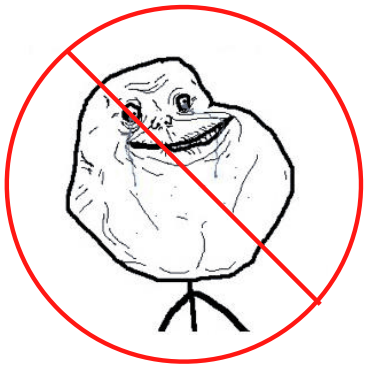

http://cs61a.org/office-hours.html

Iteration Example

The Fibonacci Sequence

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The Fibonacci Sequence

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987

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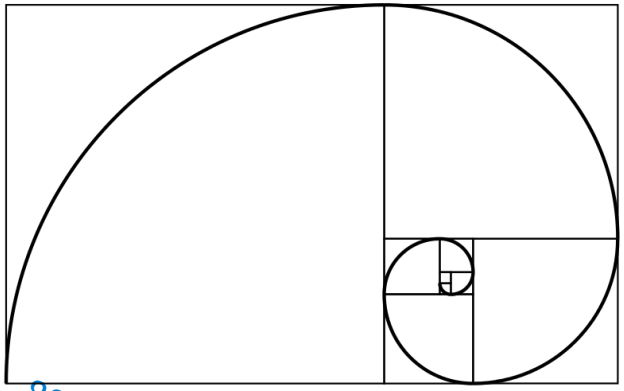

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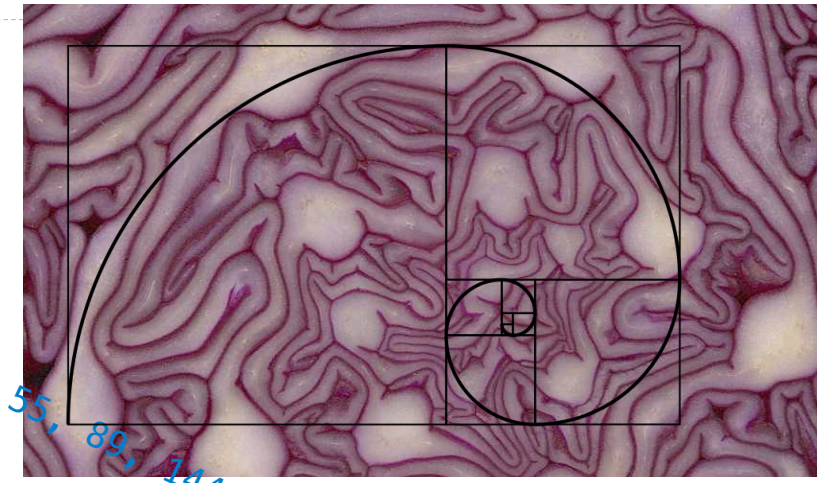

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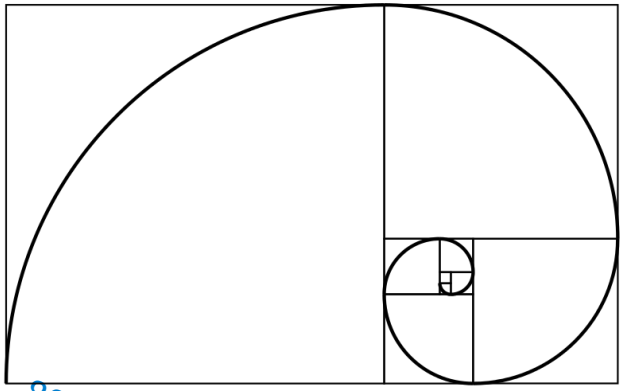

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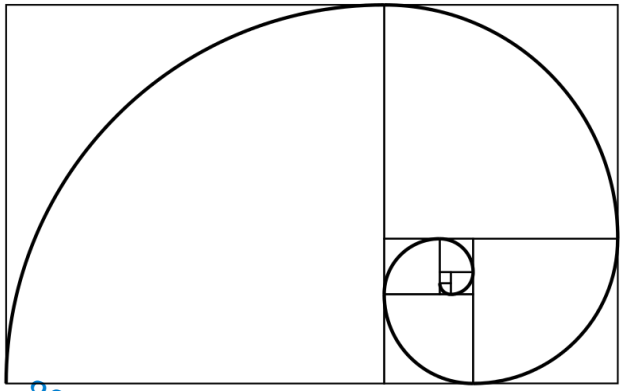

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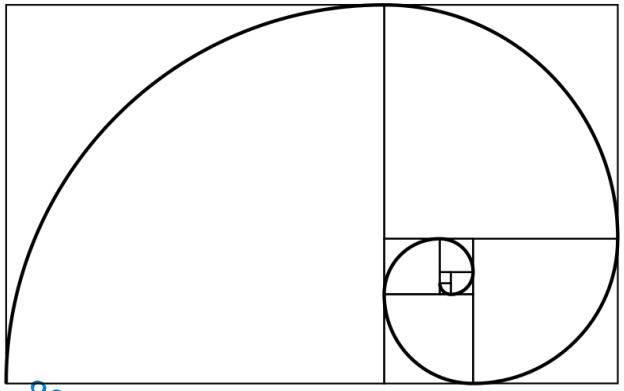

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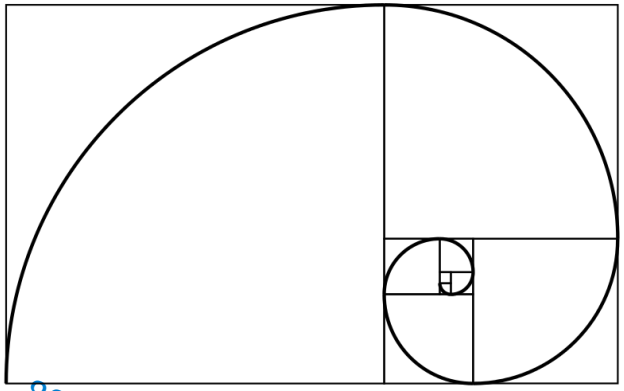

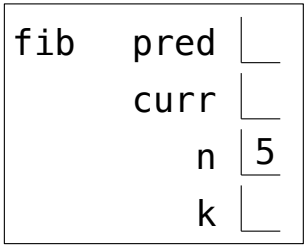


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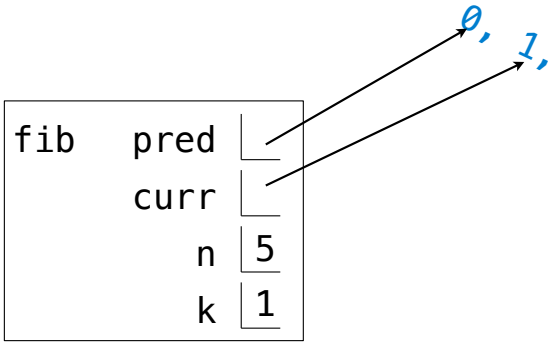

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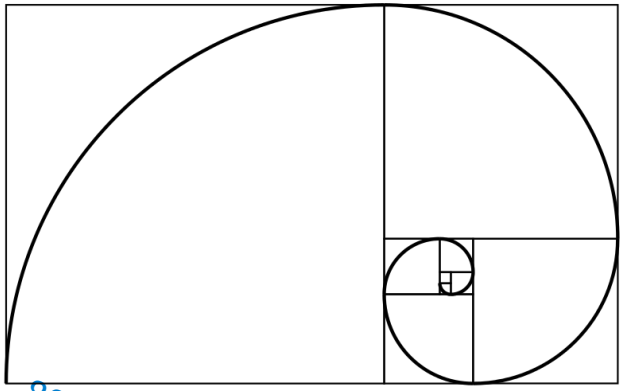

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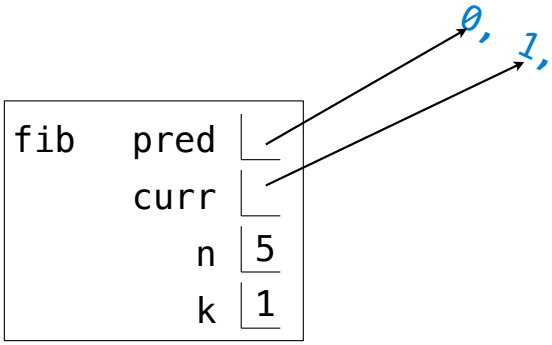

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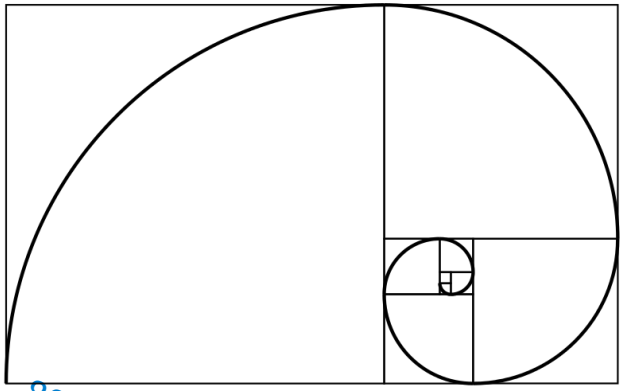

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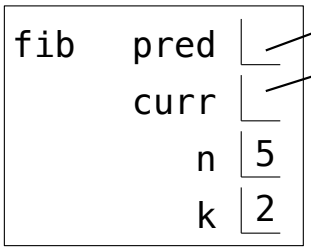

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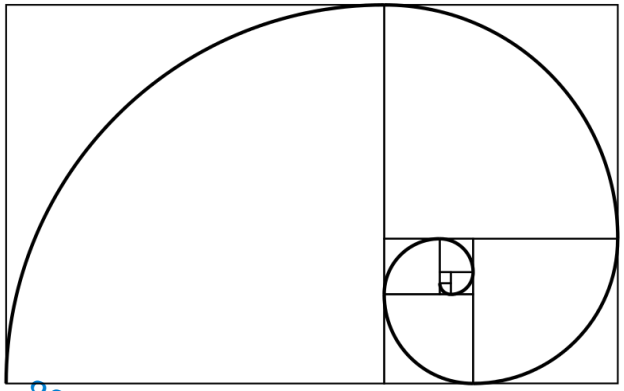

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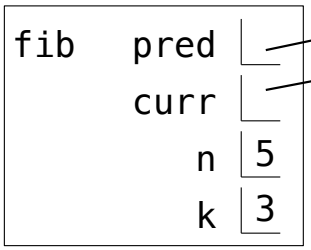

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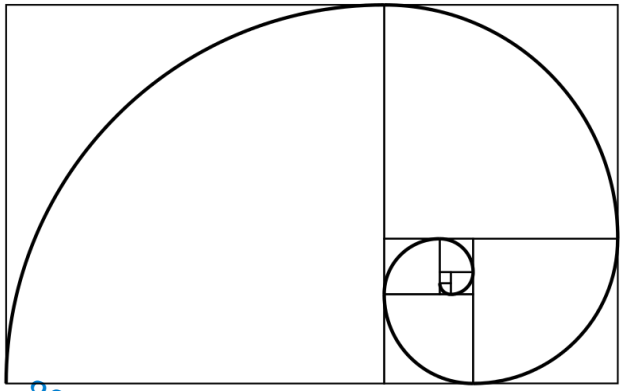

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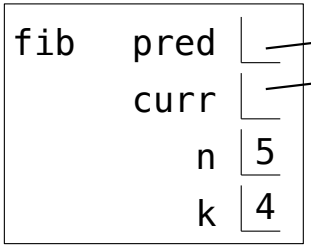

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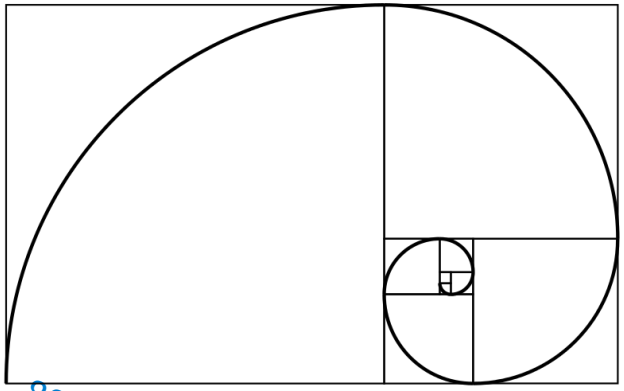

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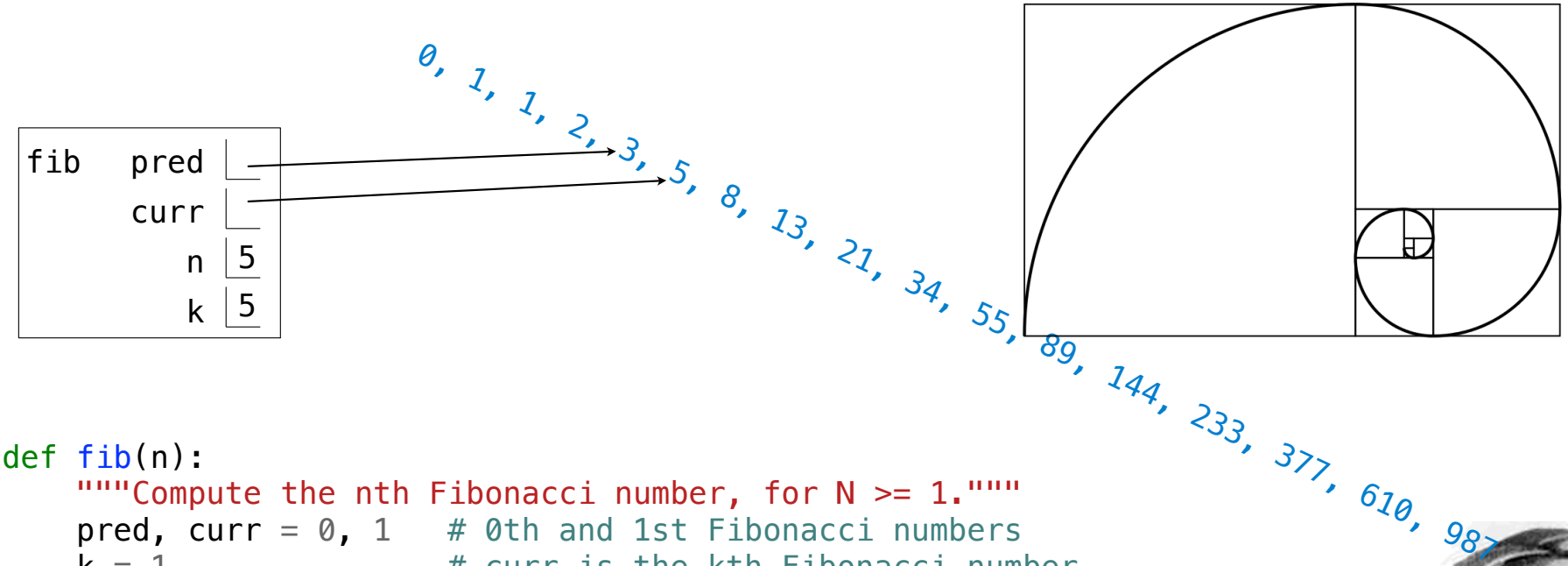


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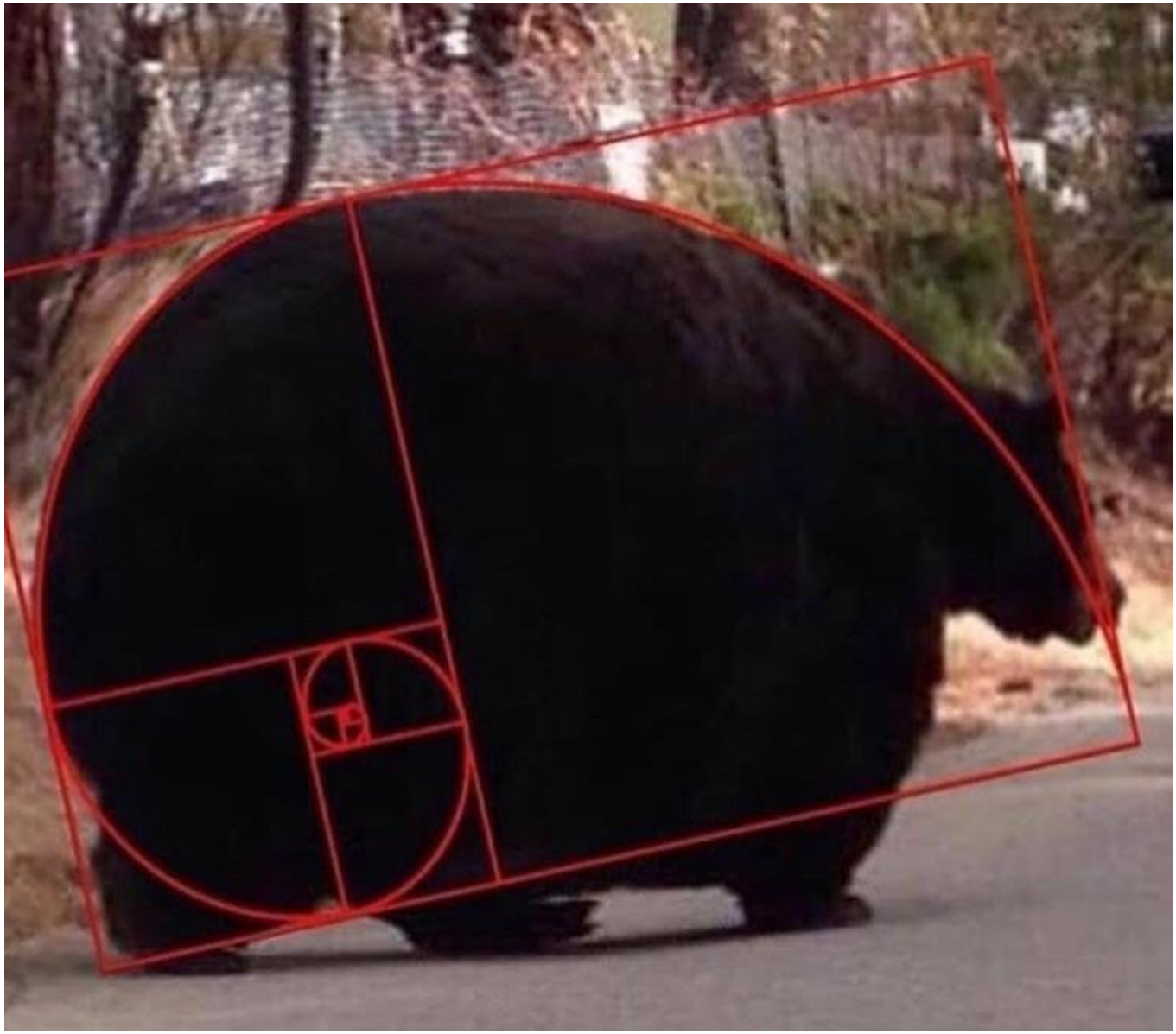
Go Bears!



Designing Functions

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square returns the square of x

A Guide to Designing Function

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(Demo)

Generalization

Generalizing Patterns with Arguments

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```
Regular geometric shapes relate length and area.
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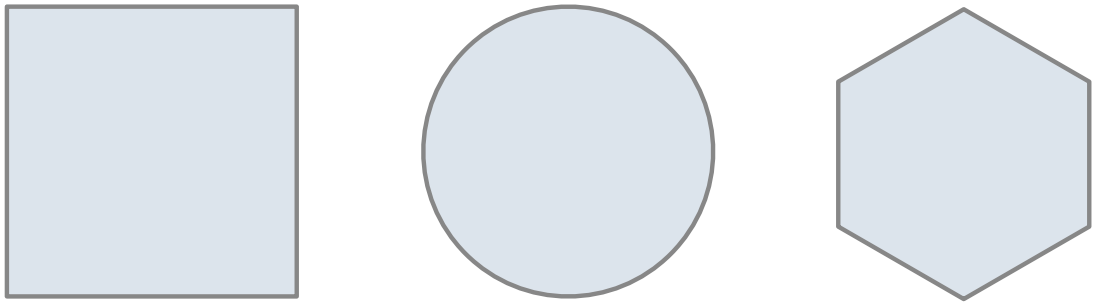
Regular geometric shapes relate length and area.

Shape:



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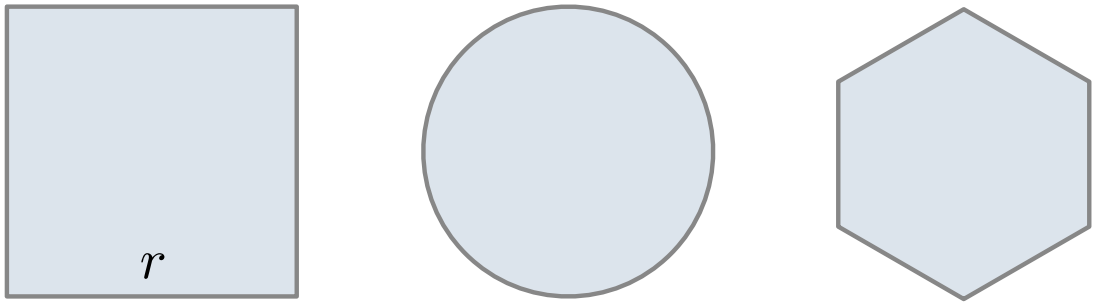

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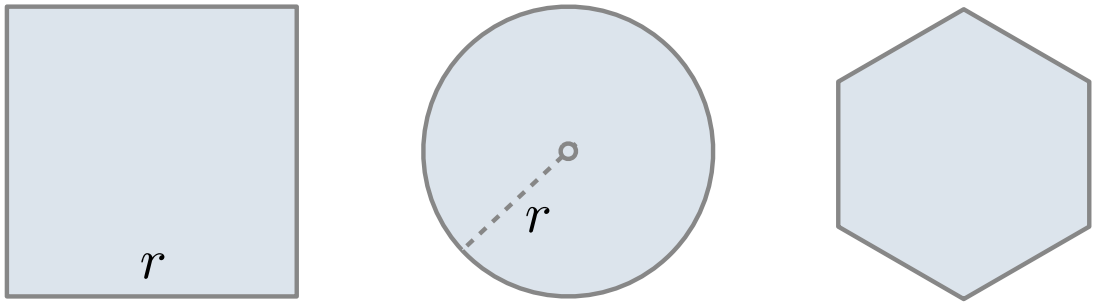

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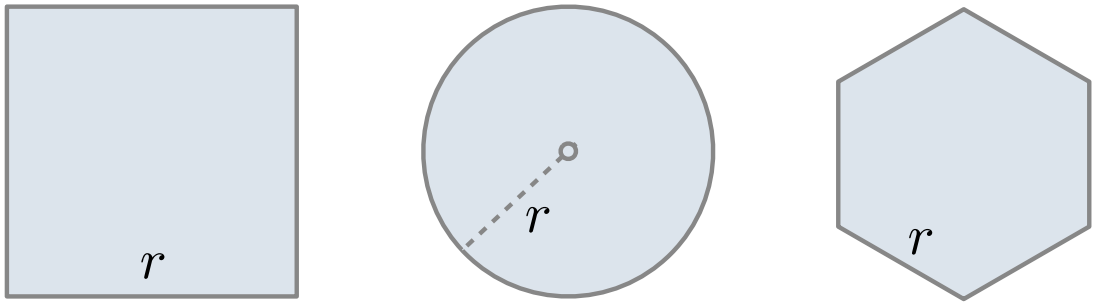

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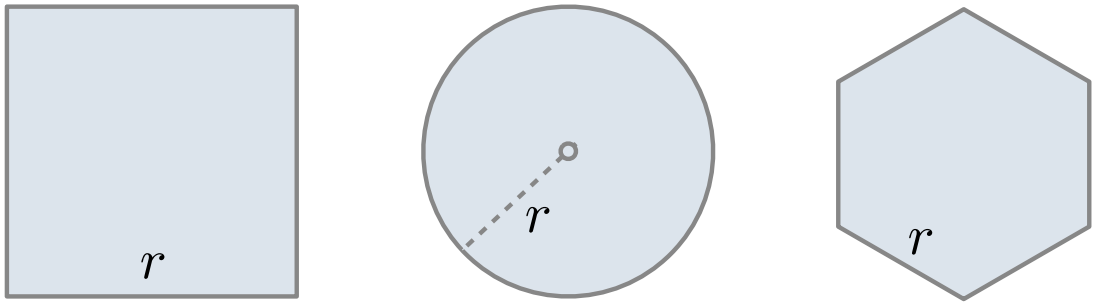

Area:

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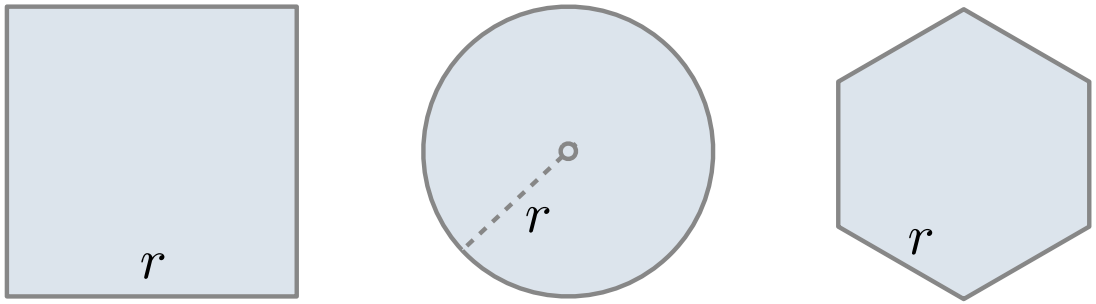

Area: r^2

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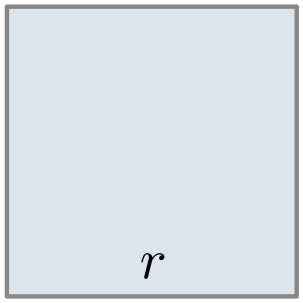

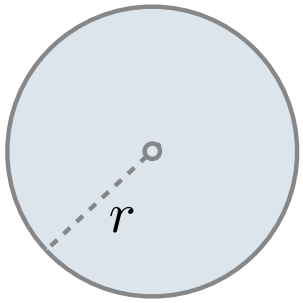

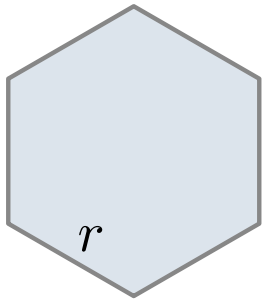

Area: r^2 $\pi \cdot r^2$

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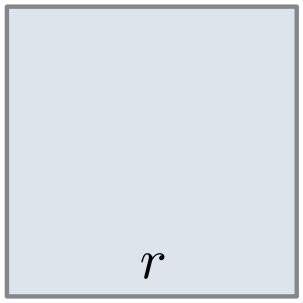
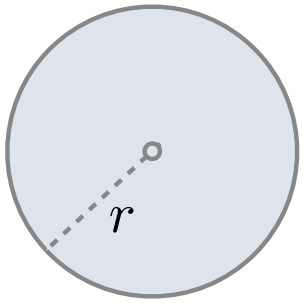
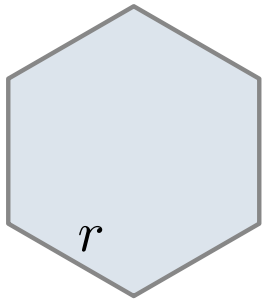
Shape:			
Area:	r^2	$\pi \cdot r^2$	$\frac{3\sqrt{3}}{2} \cdot r^2$

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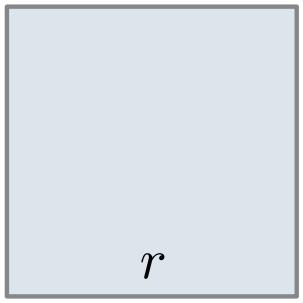
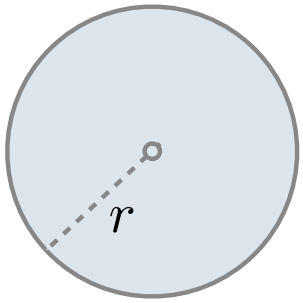
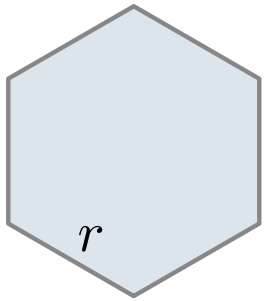
Shape:			
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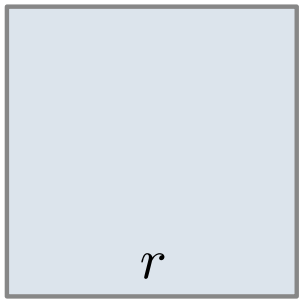
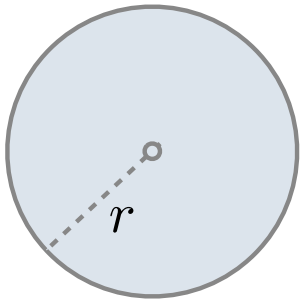
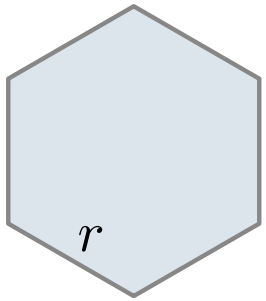
Shape:			
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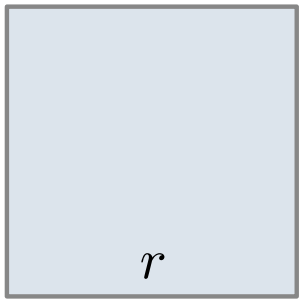
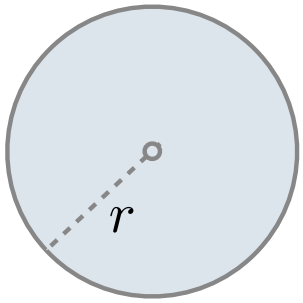
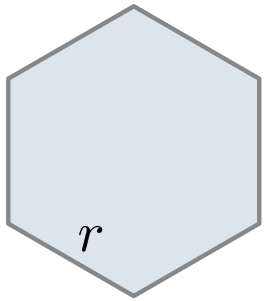
Shape:			
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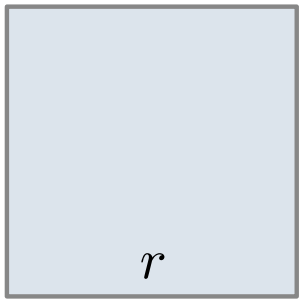

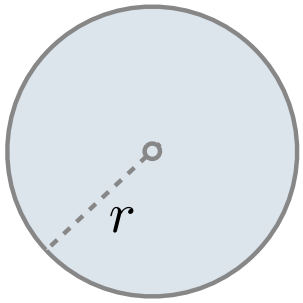

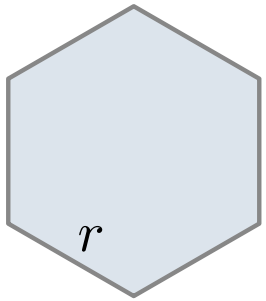

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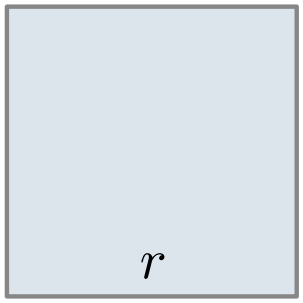

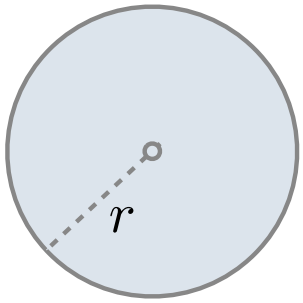

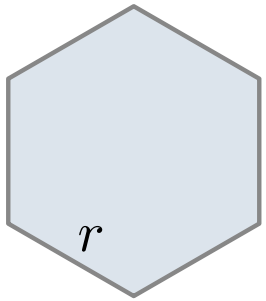

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Finding common structure allows for shared implementation

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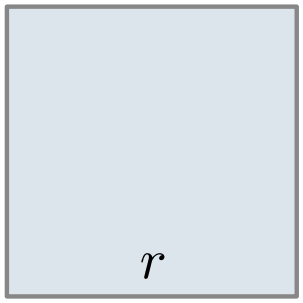

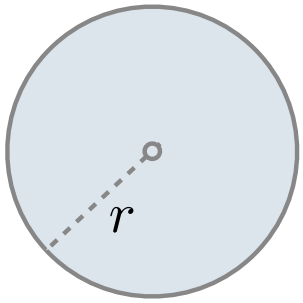

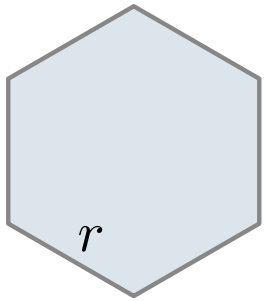

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(Demo)

Higher-Order Functions

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$$\sum_{k=1}^{5} \frac{8}{(4k-3)\cdot(4k-1)} = \frac{8}{3} + \frac{8}{35} + \frac{8}{99} + \frac{8}{195} + \frac{8}{323} \qquad = 3.04$$

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(Demo)

Summation Example

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def cube(k):
    return pow(k, 3)

def summation(n, term):
    """Sum the first n terms of a sequence.

    >>> summation(5, cube)
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    total, k = 0, 1
    while k <= n:
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Functions as Return Values

(Demo)

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def make_adder(n):
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A def statement within another def statement

Can refer to names in the enclosing function

Call Expressions as Operator Expressions

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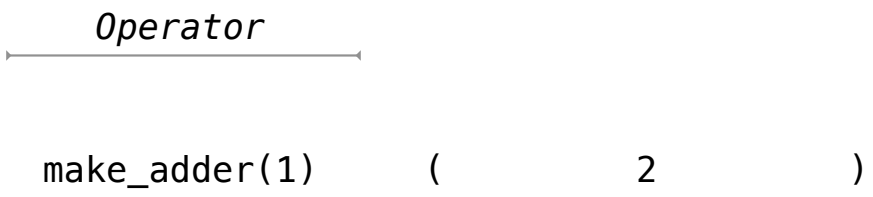
```
make_adder(1)     (         2         )
```

Call Expressions as Operator Expressions



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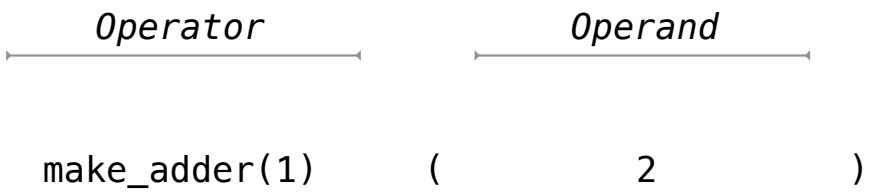

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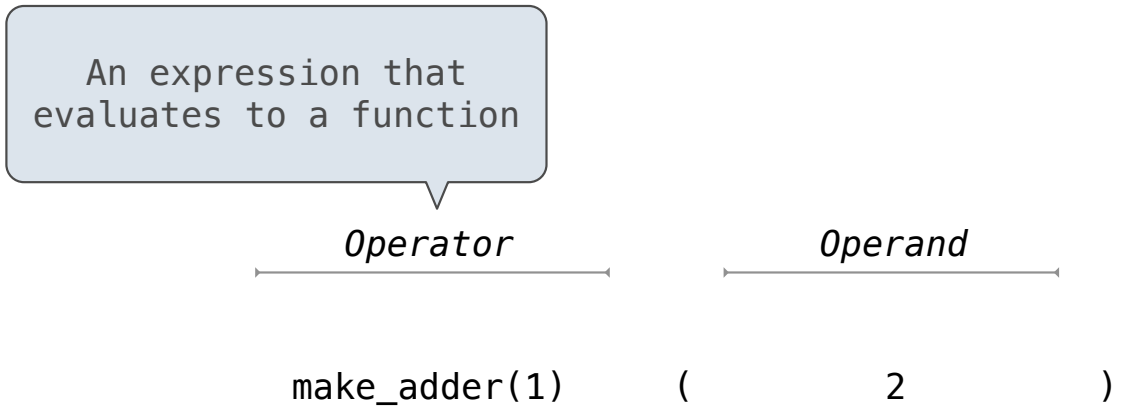

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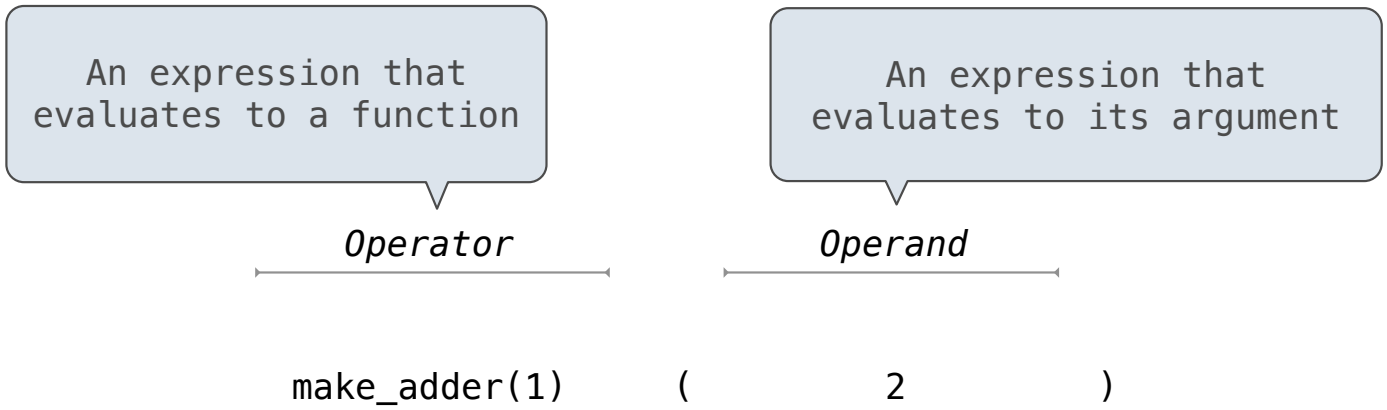

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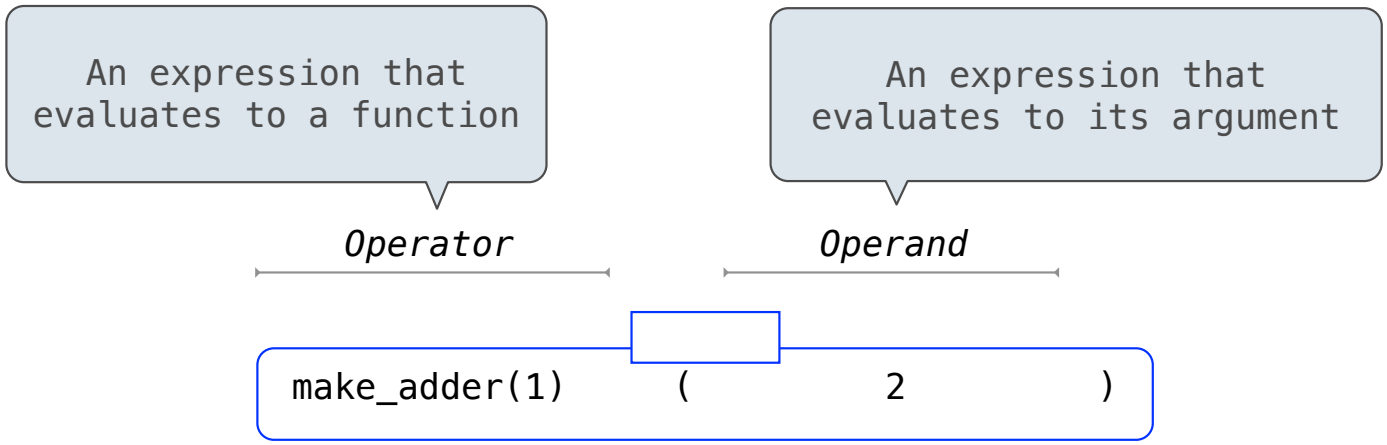

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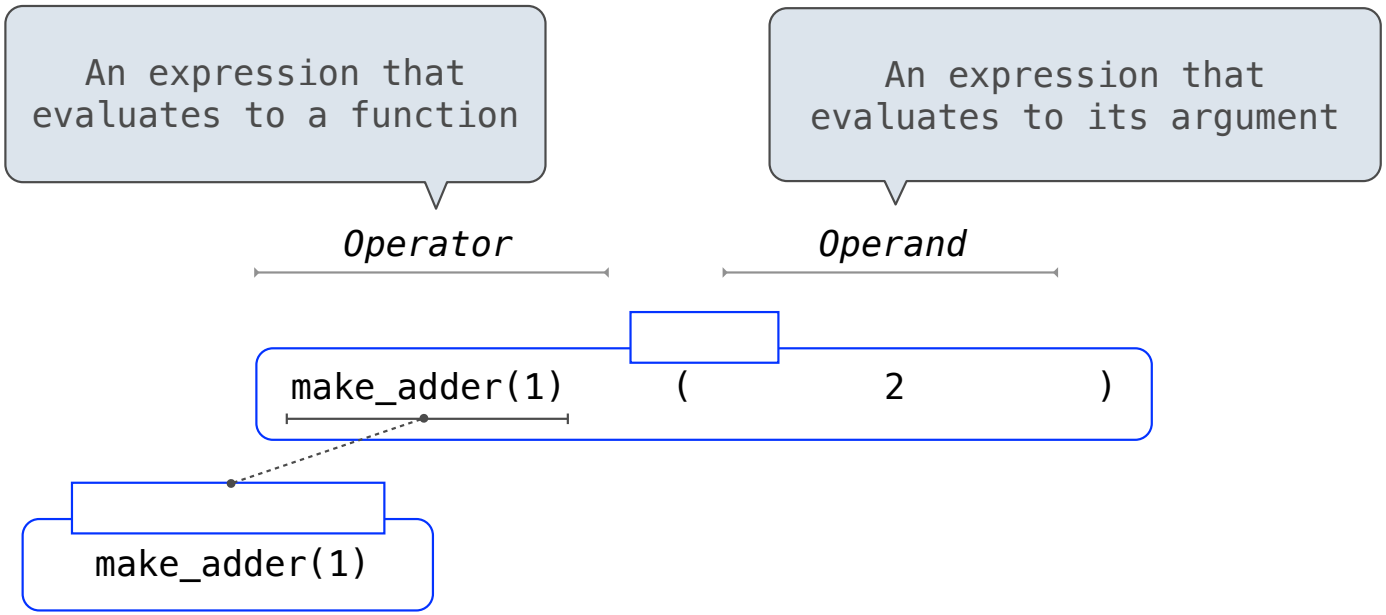

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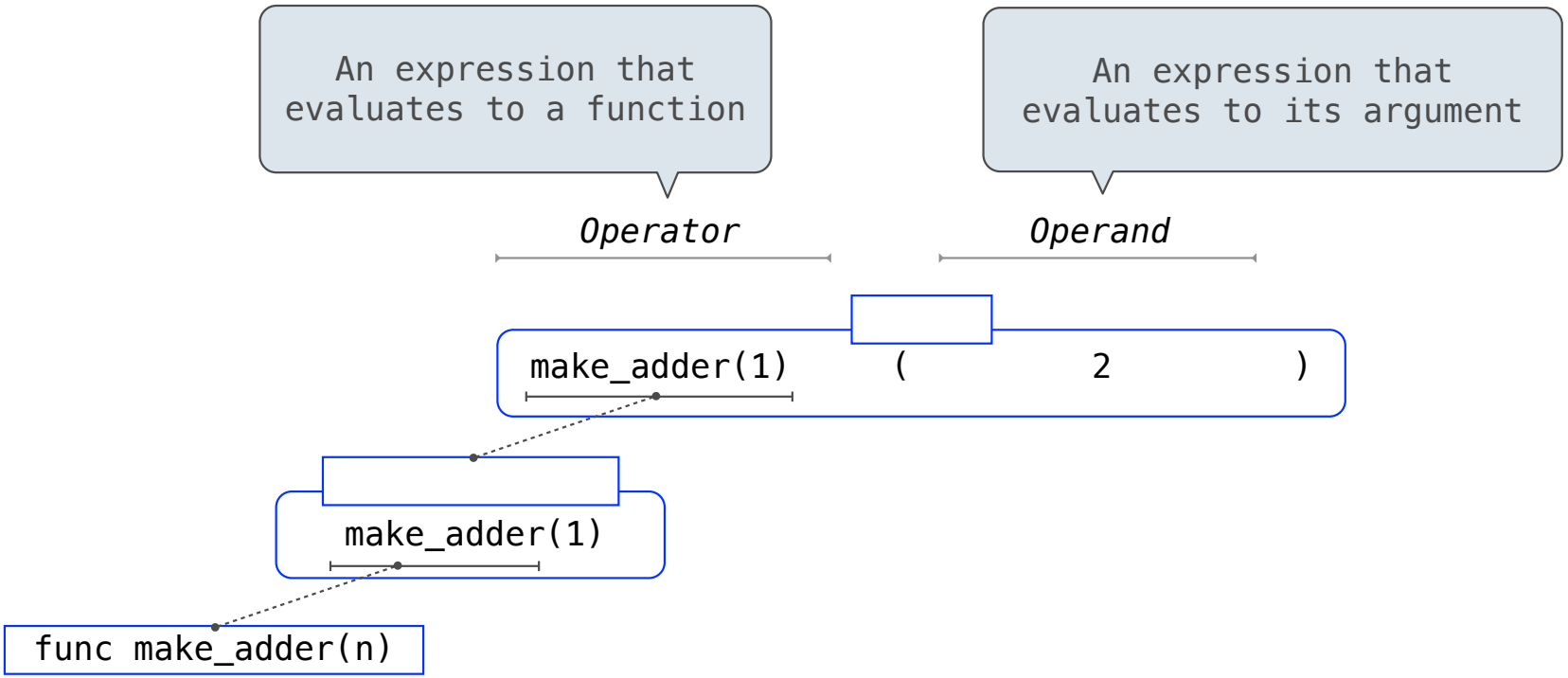

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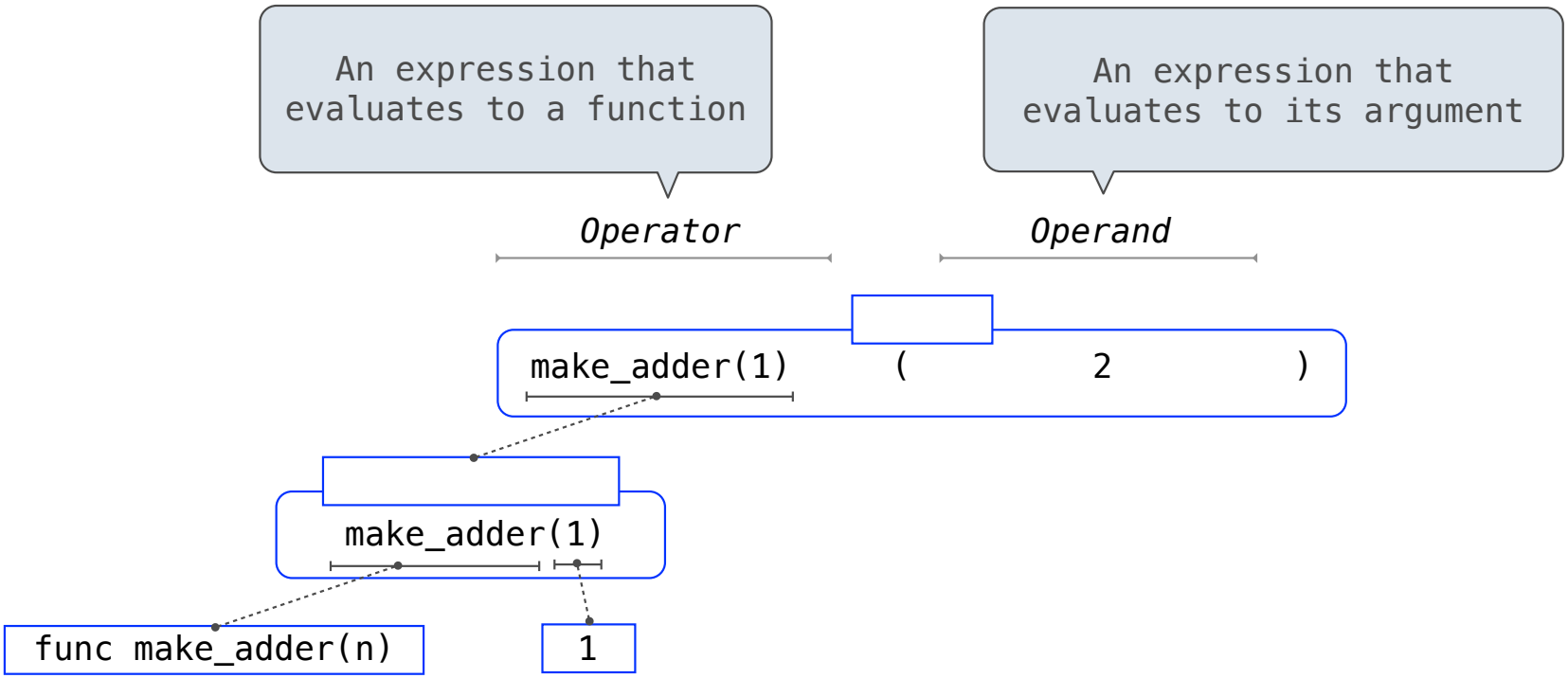

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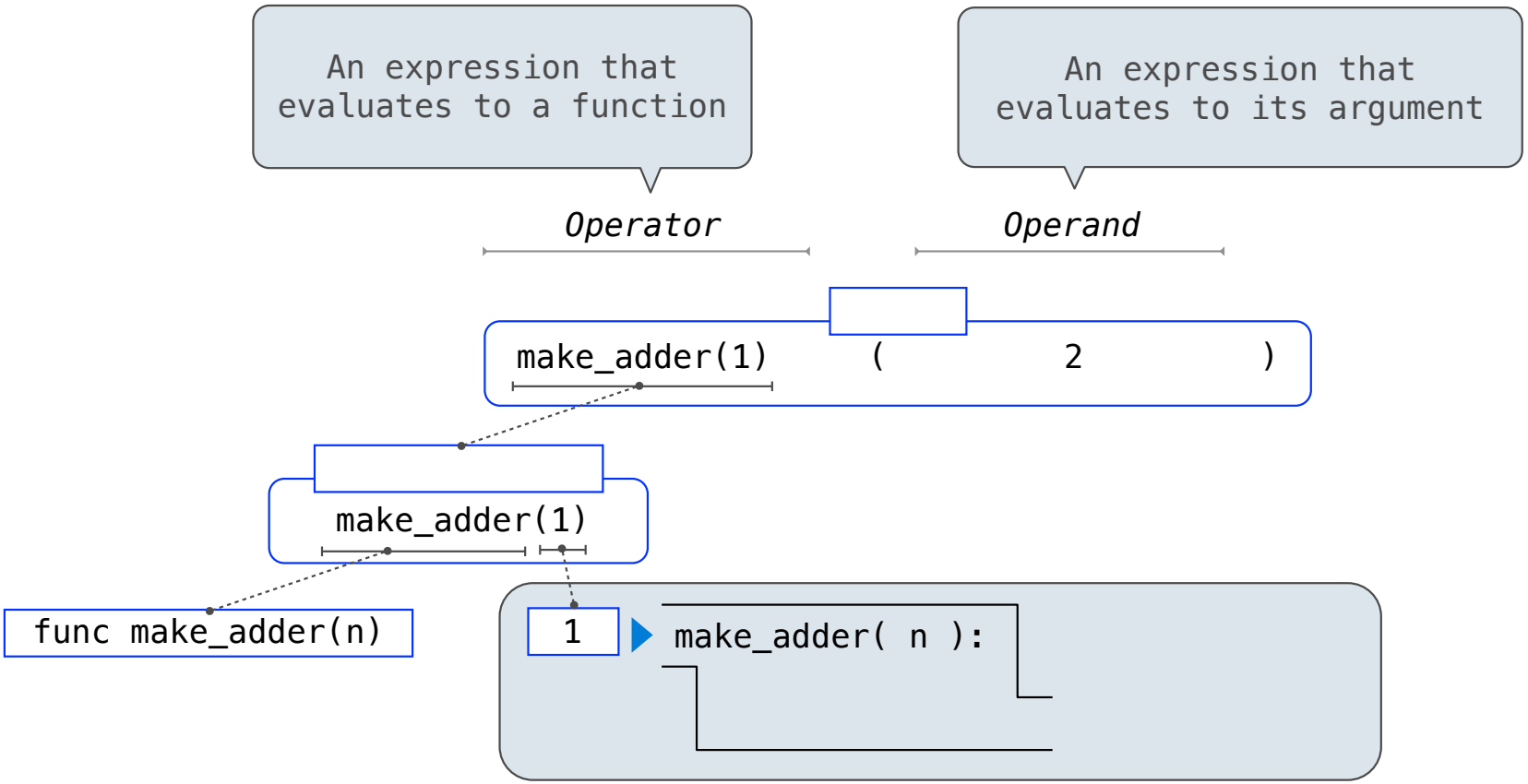

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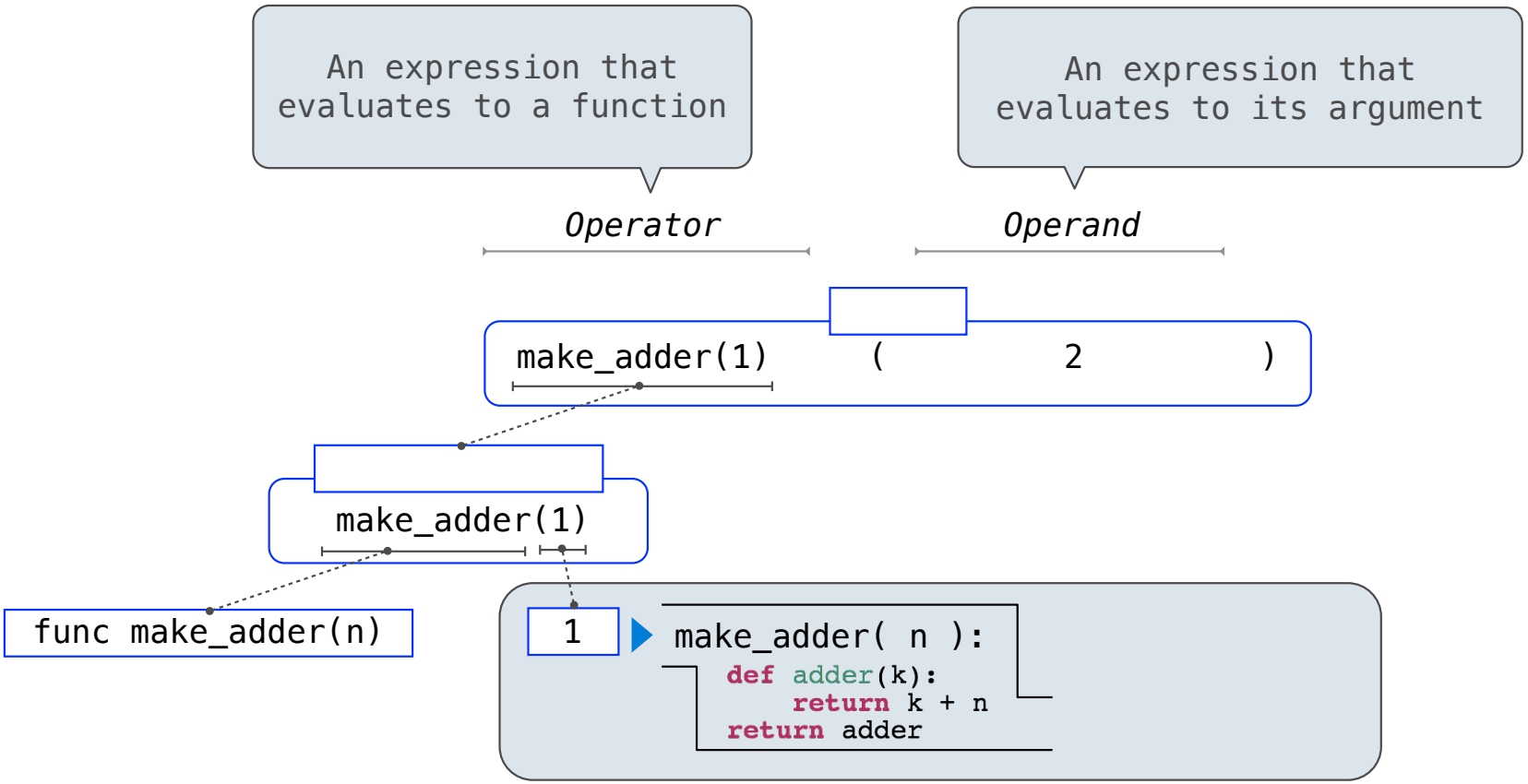

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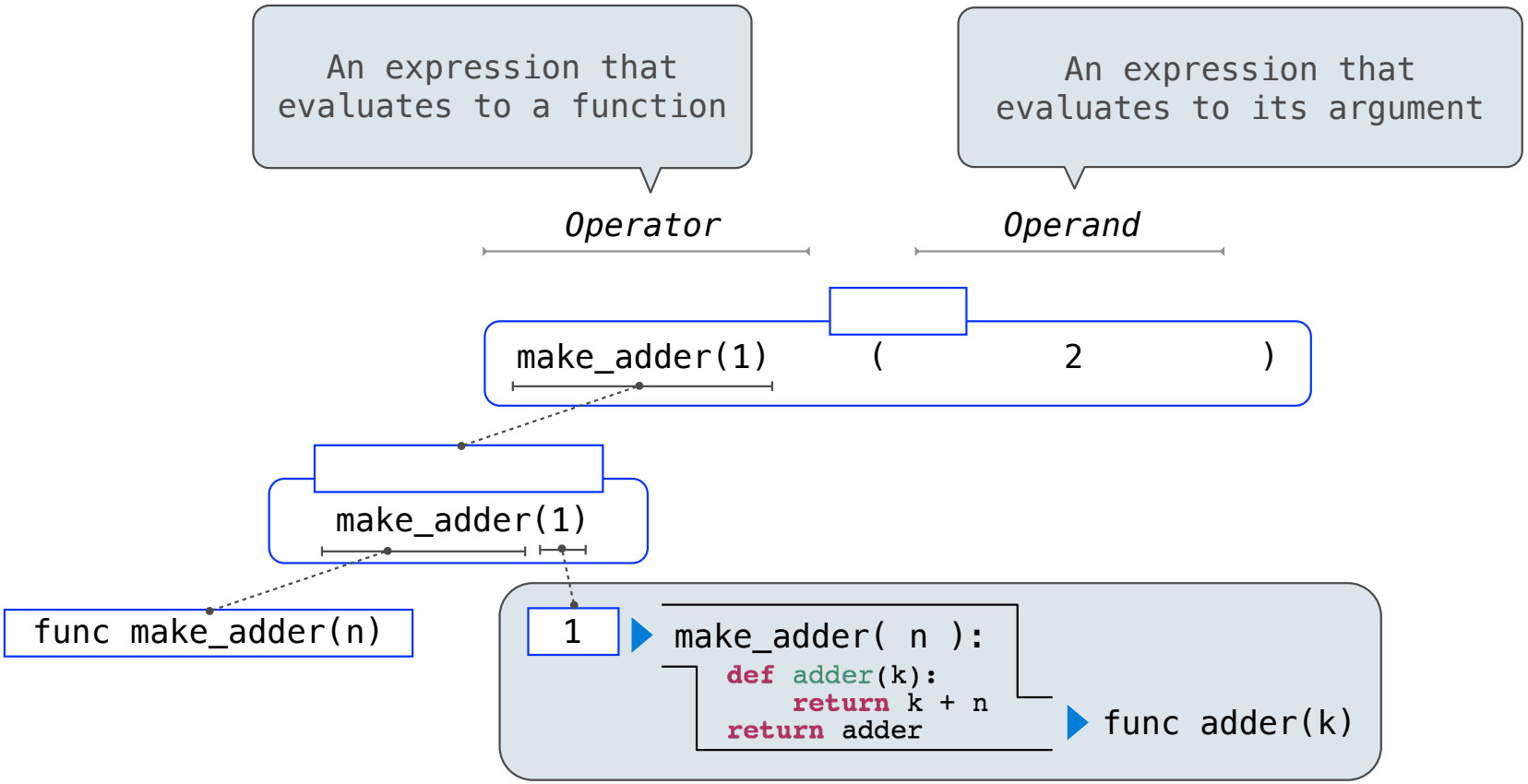

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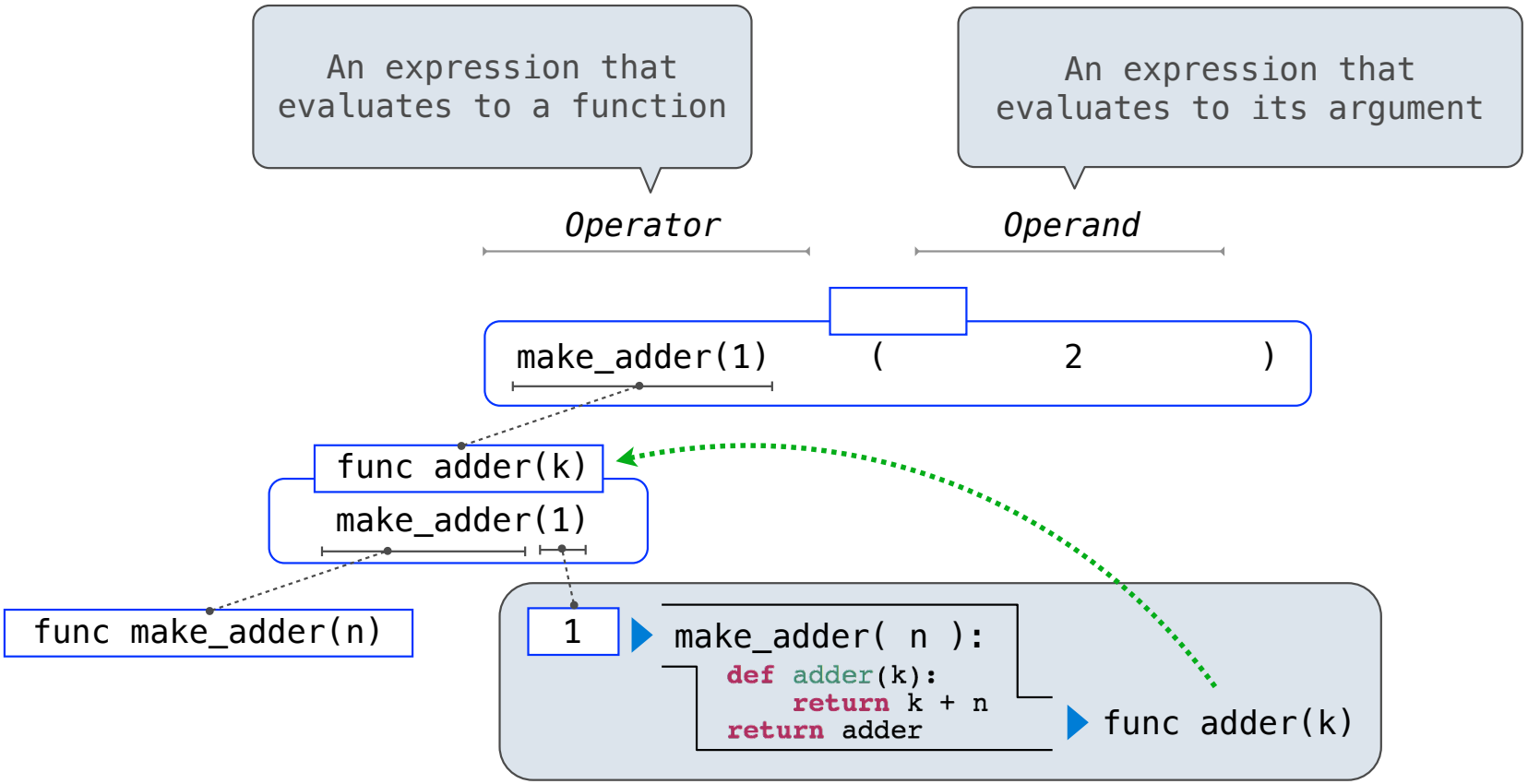

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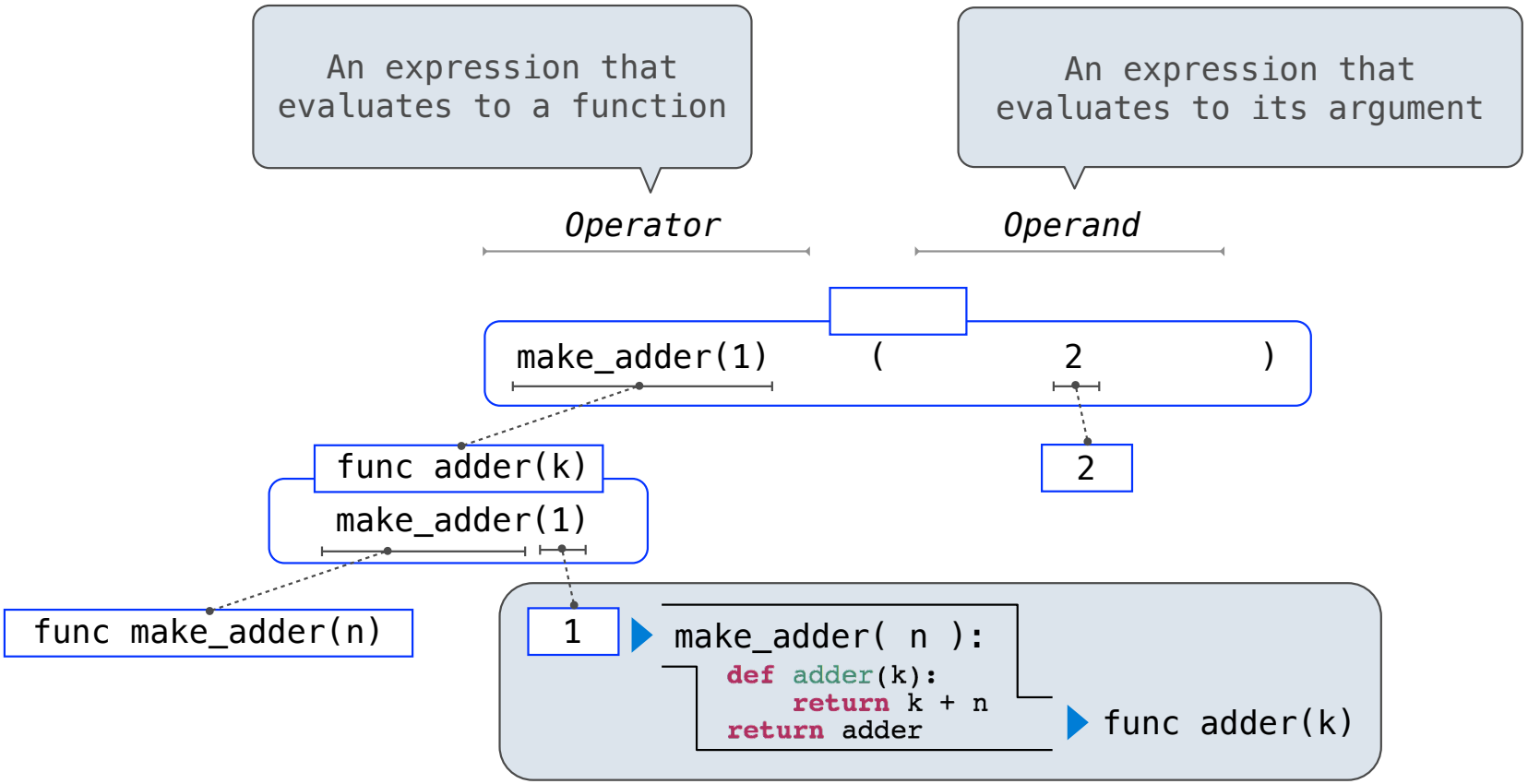

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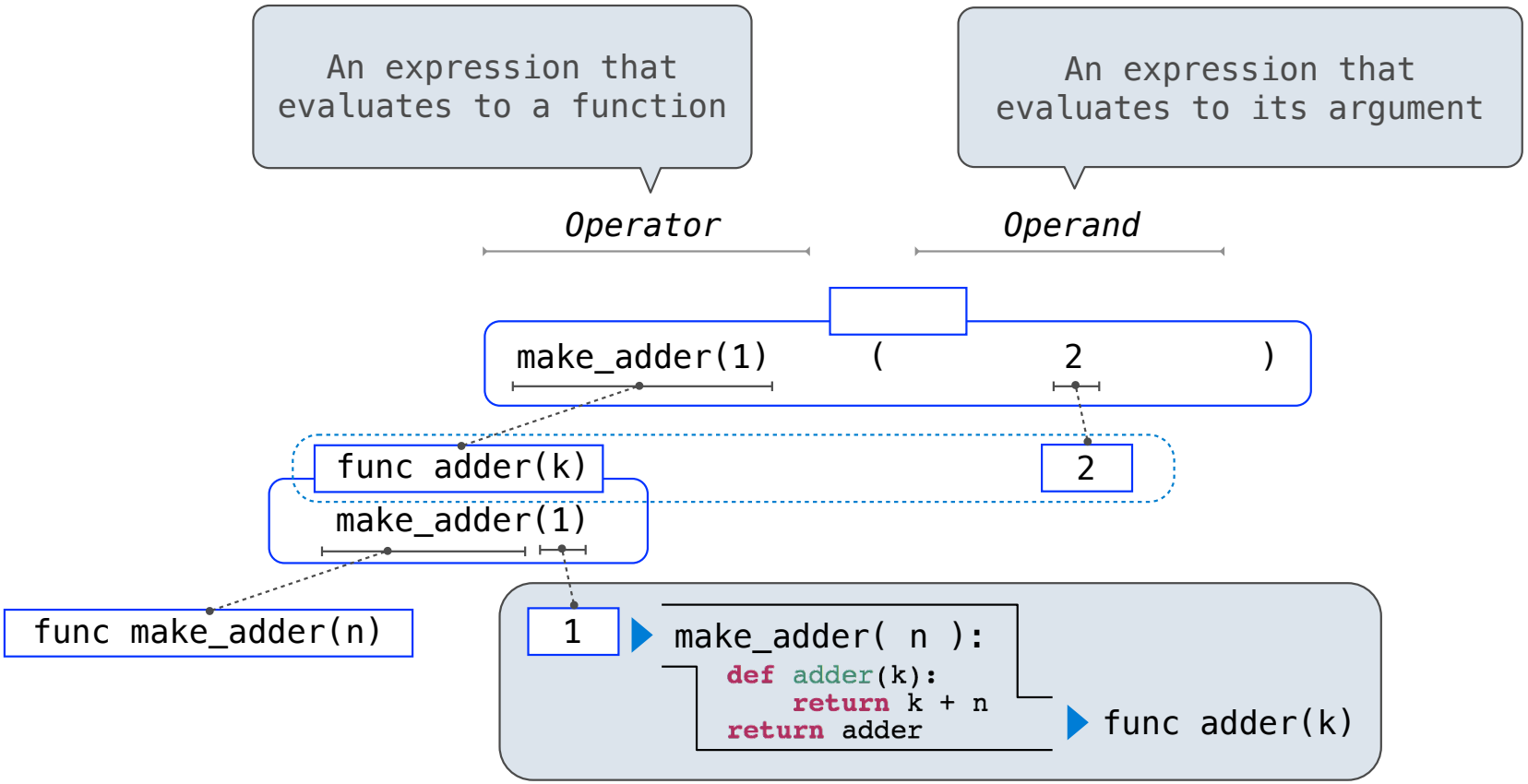

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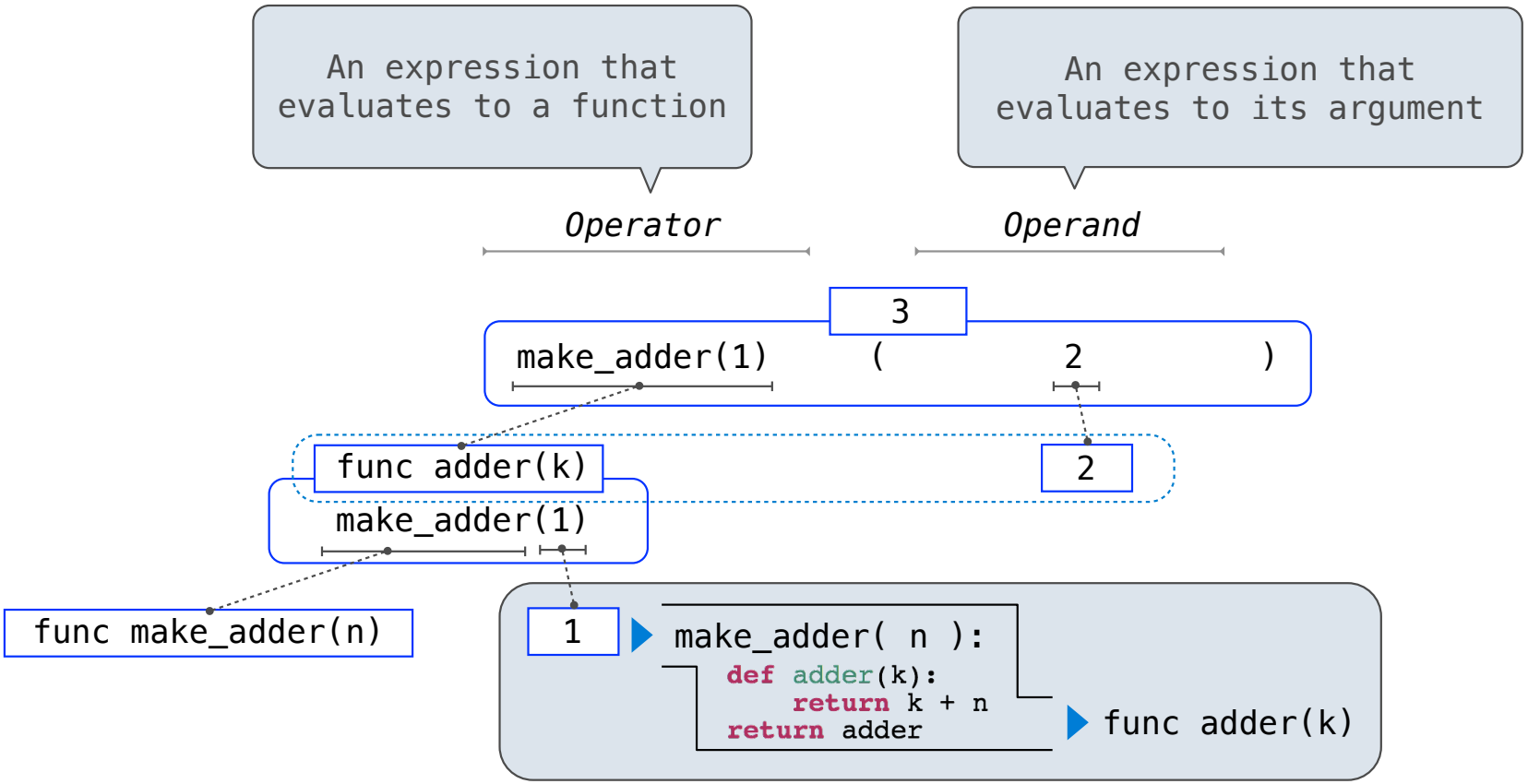

Lambda Expressions

(Demo)

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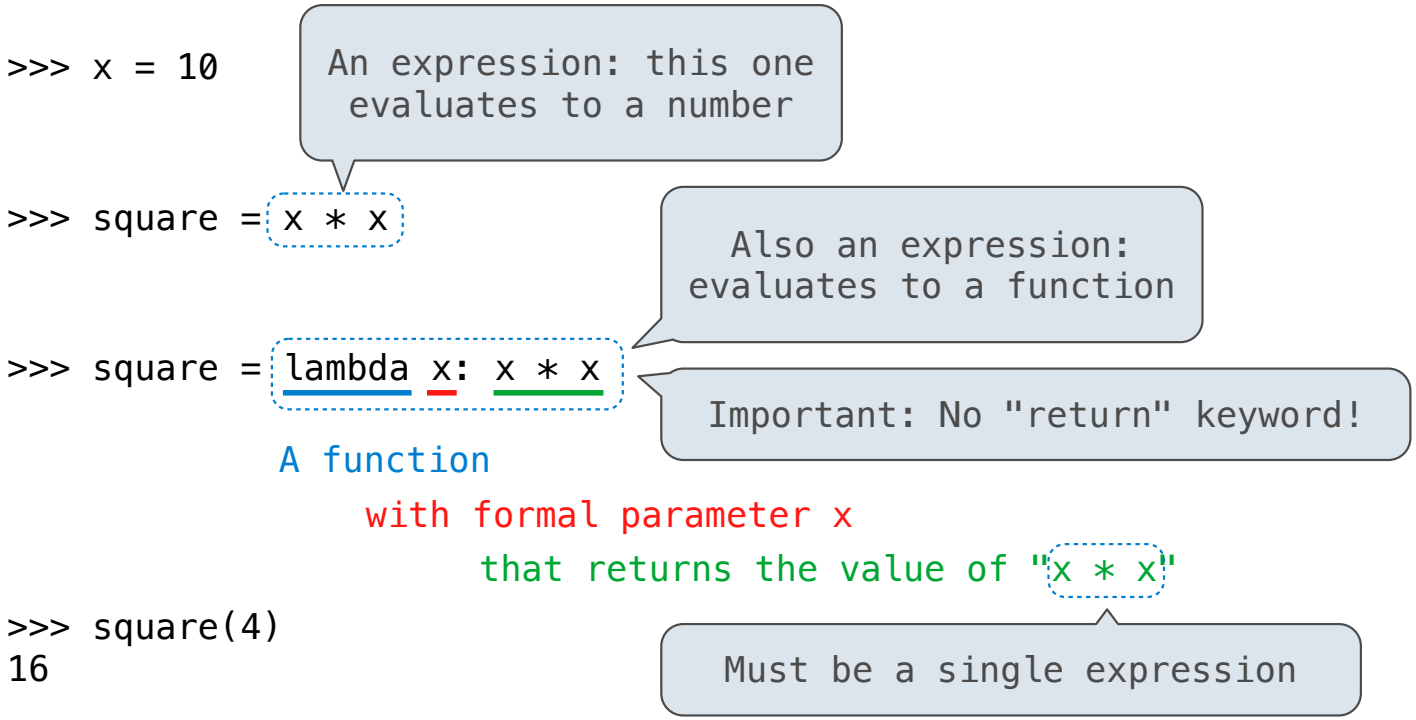

Lambda Expressions

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16
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Lambda expressions are not common in Python, but important in general

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Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

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VS

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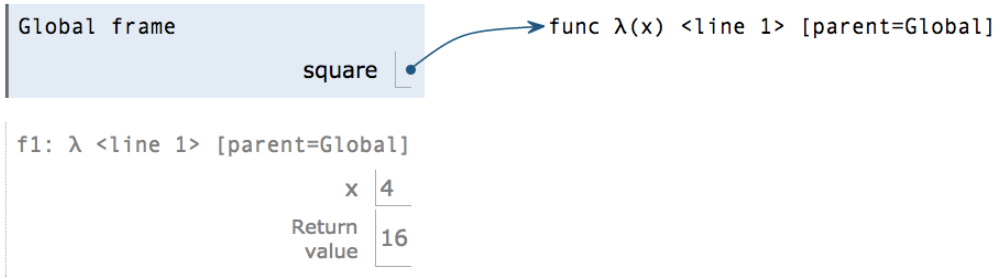

Lambda Expressions Versus Def Statements

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def square(x):
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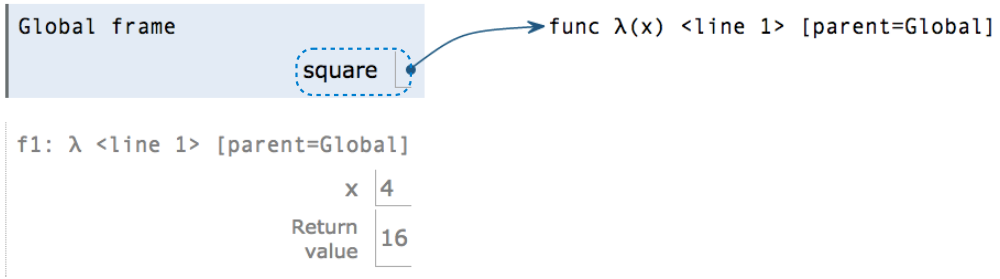

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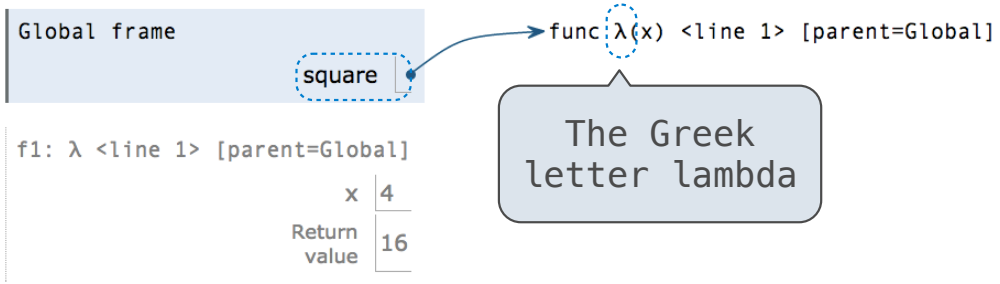

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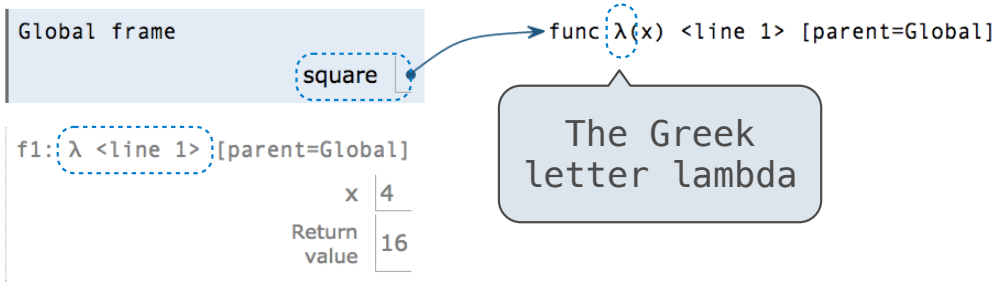

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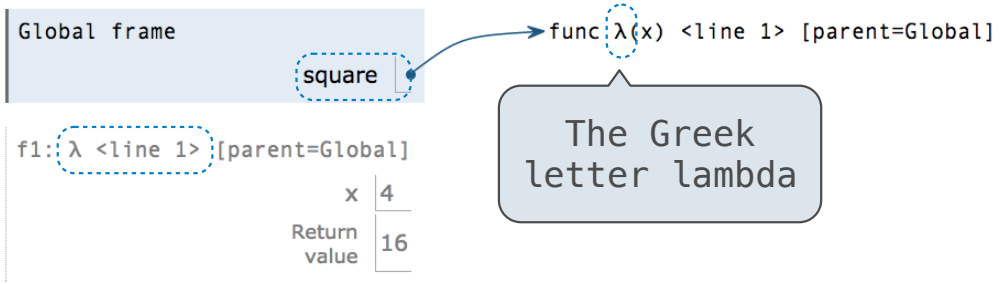

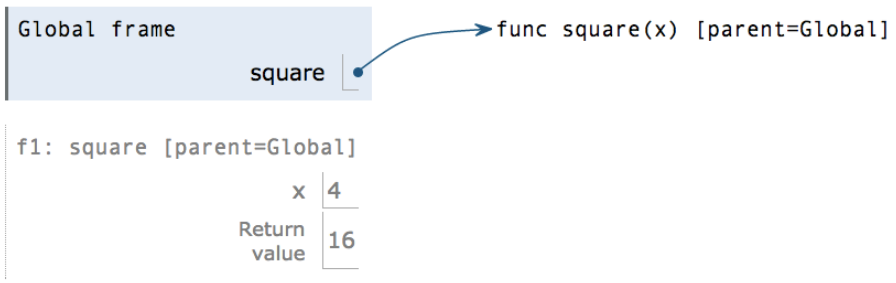

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(Demo)

Control

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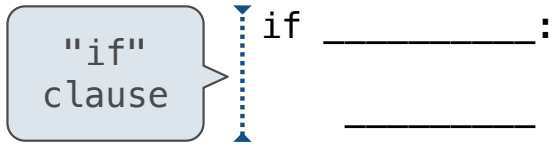

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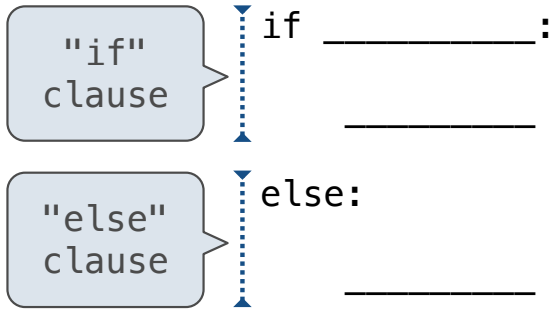

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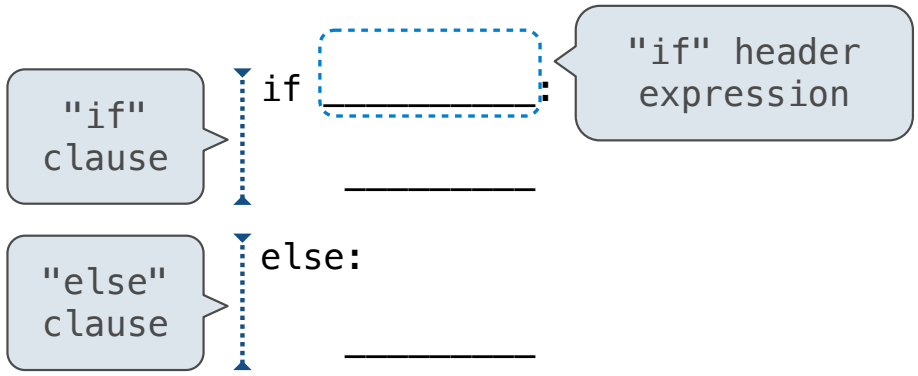

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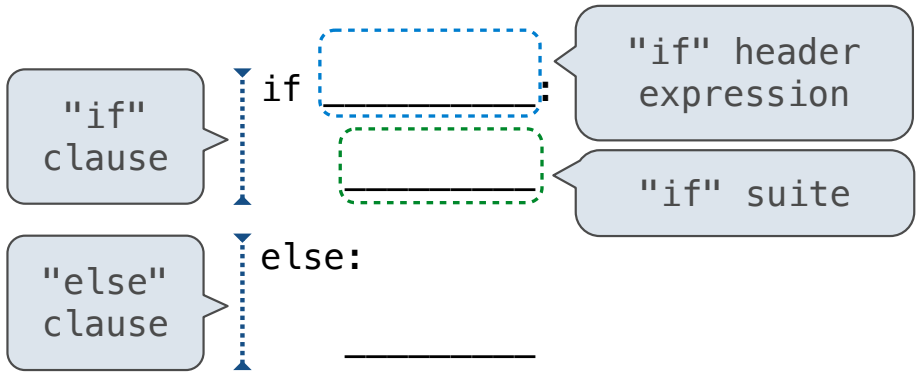

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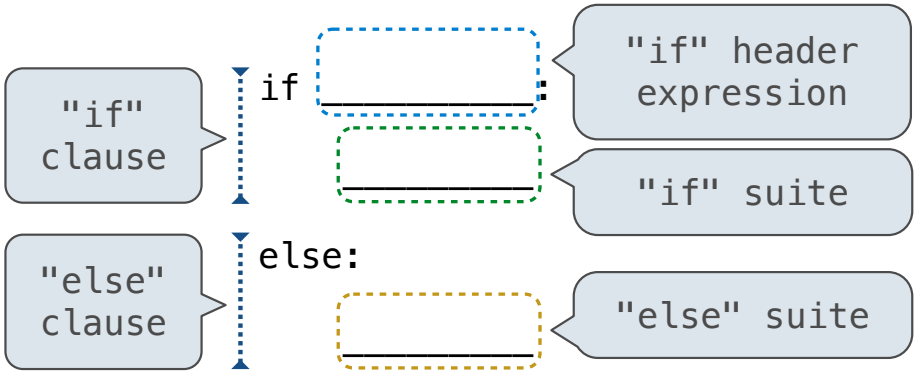

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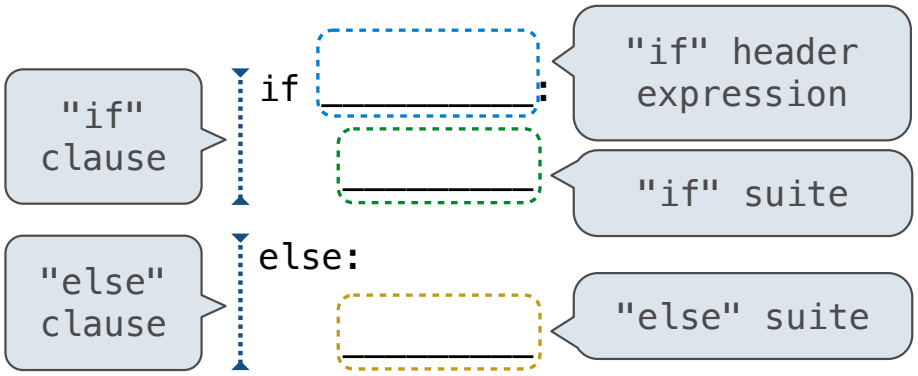

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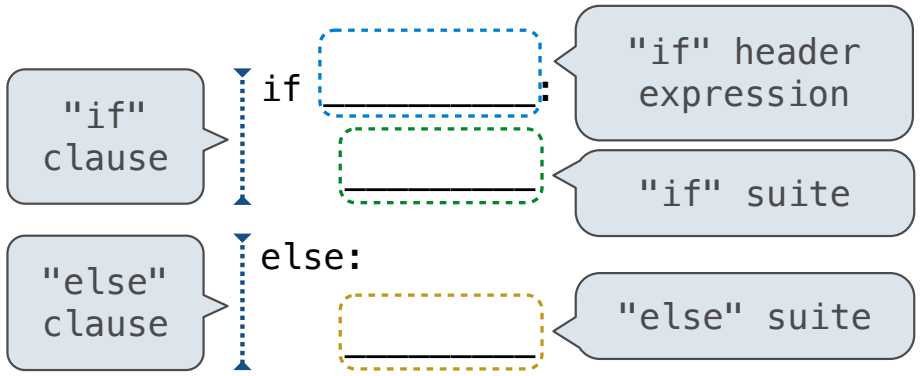

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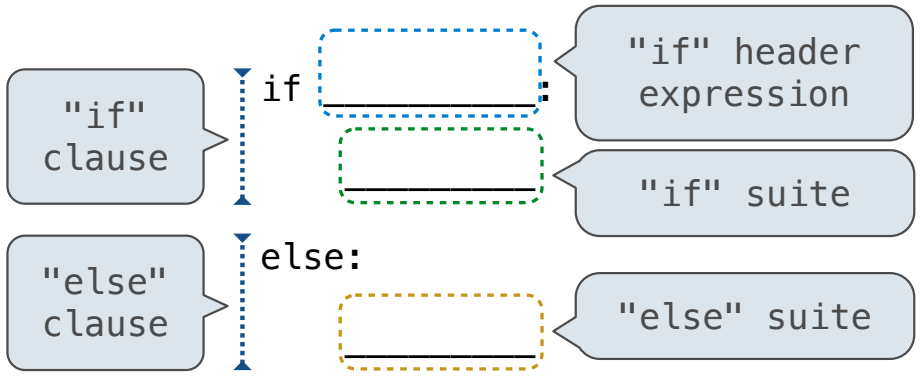

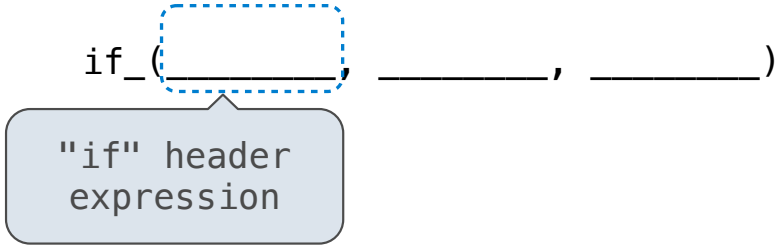

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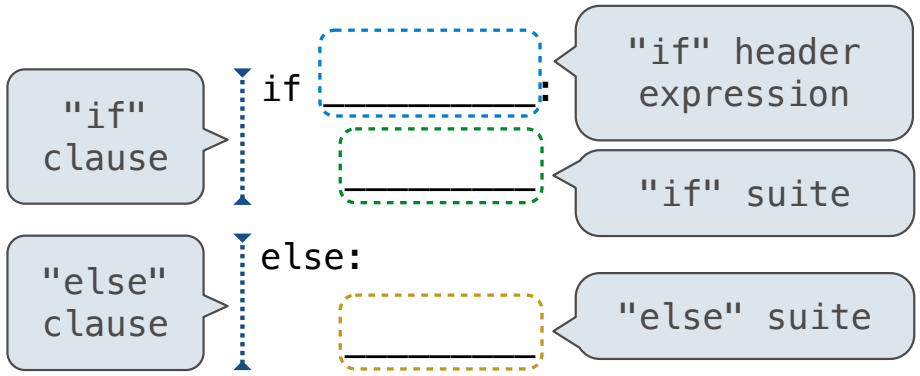

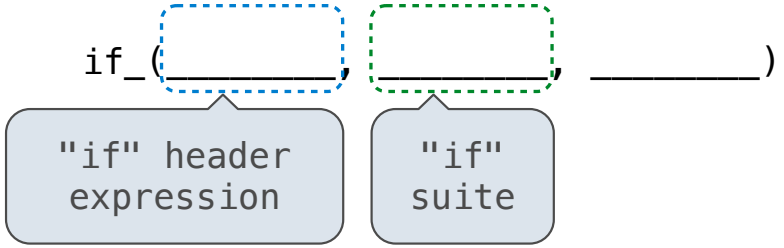

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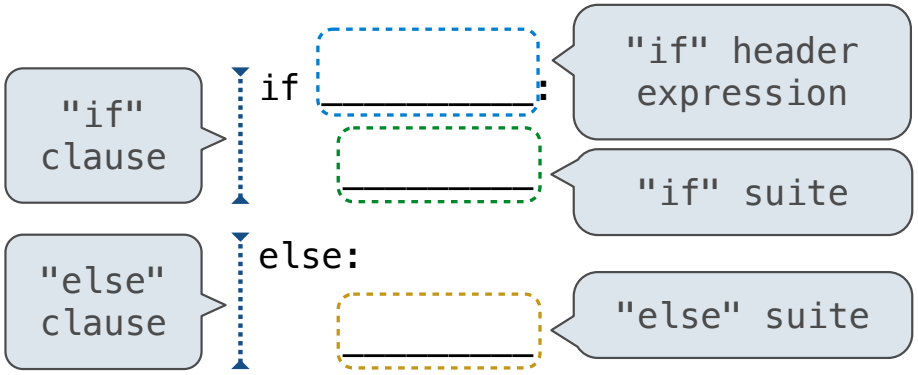

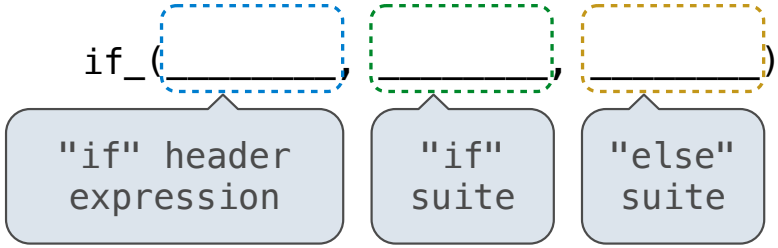

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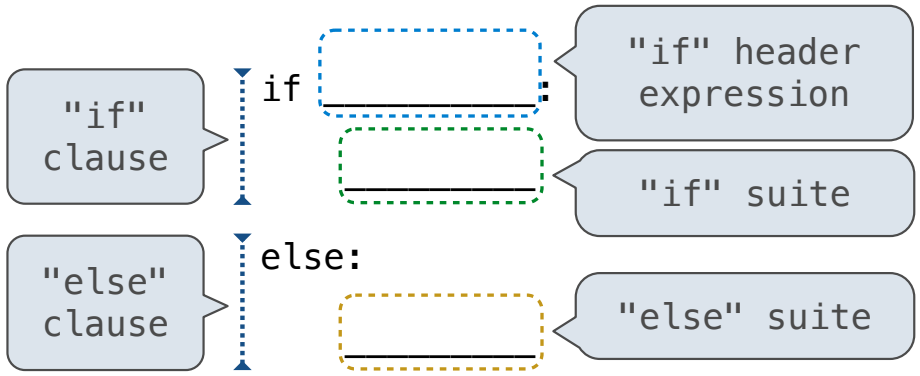

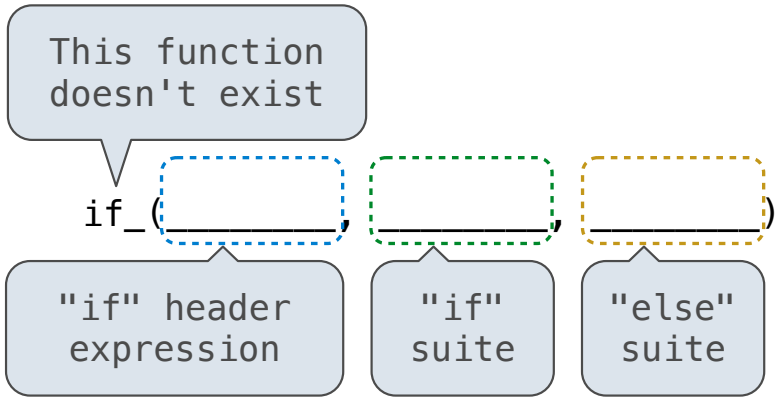

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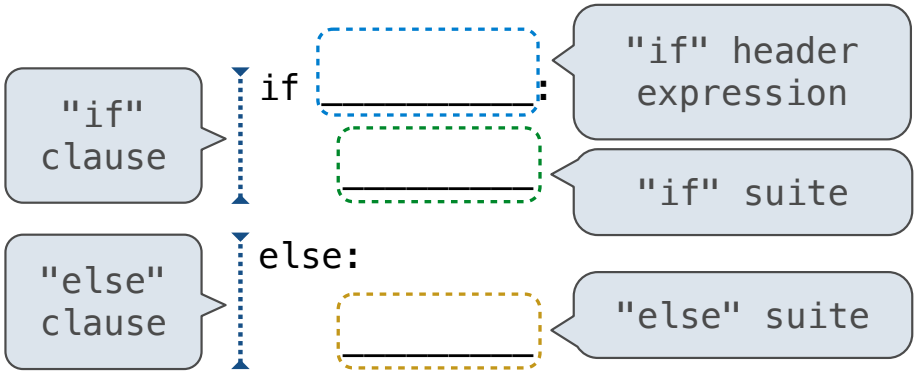

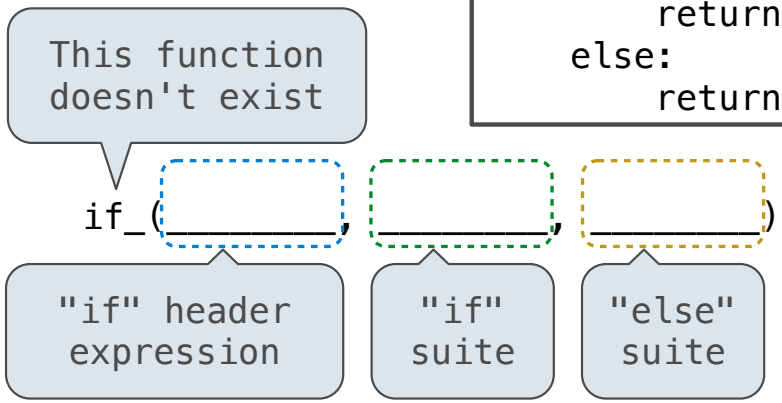

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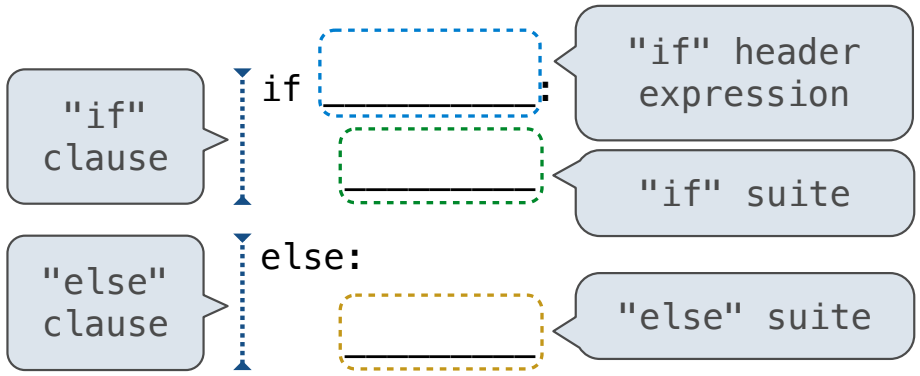

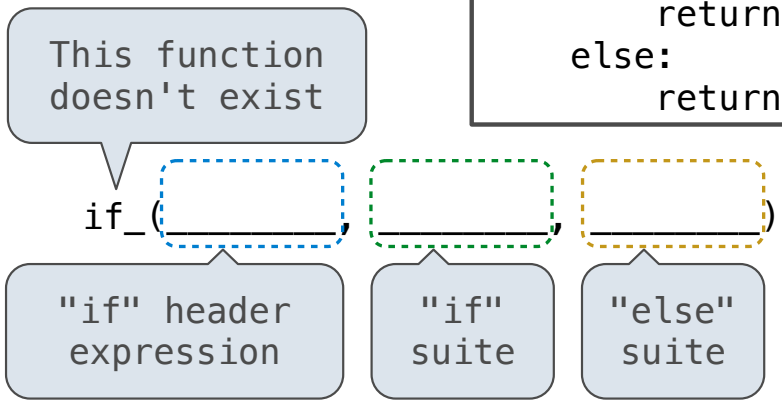

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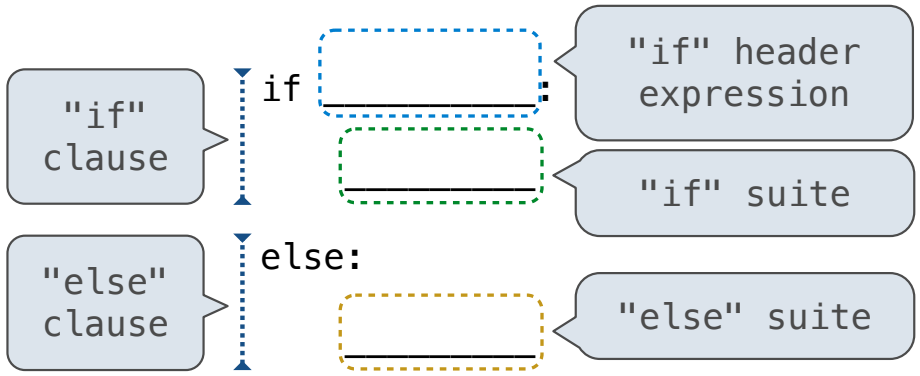

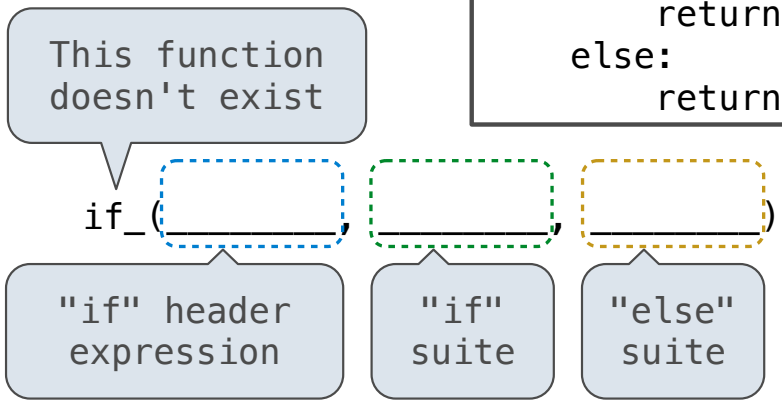

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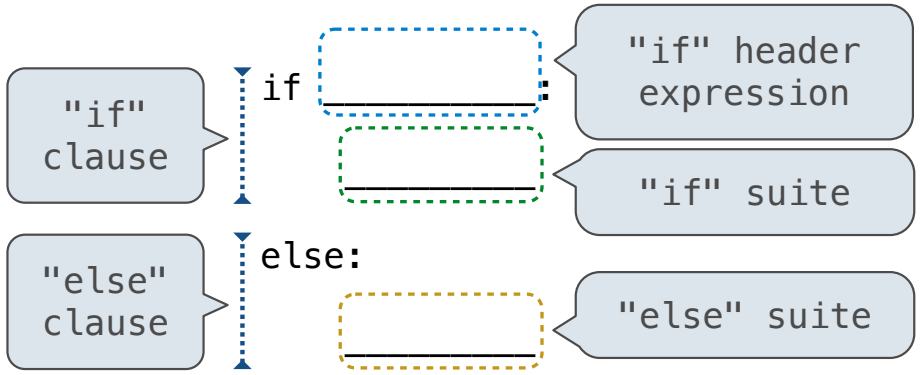

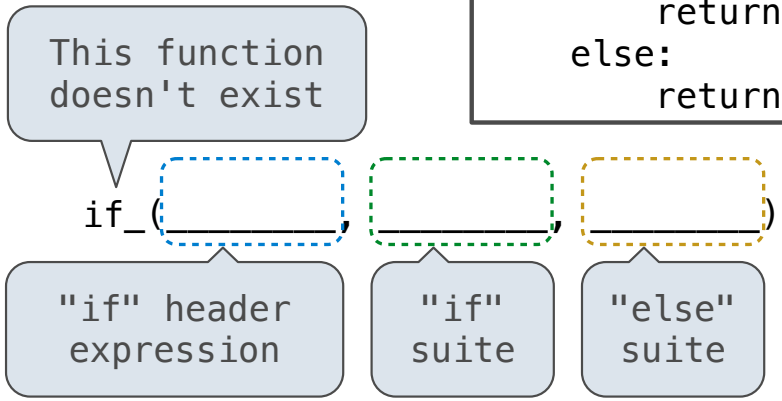

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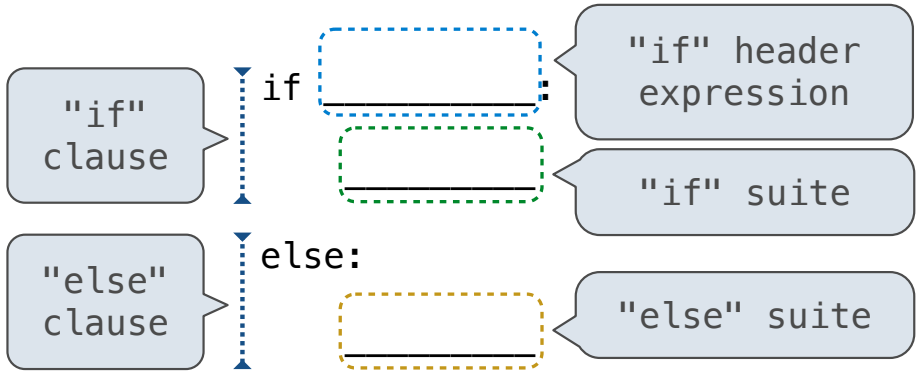

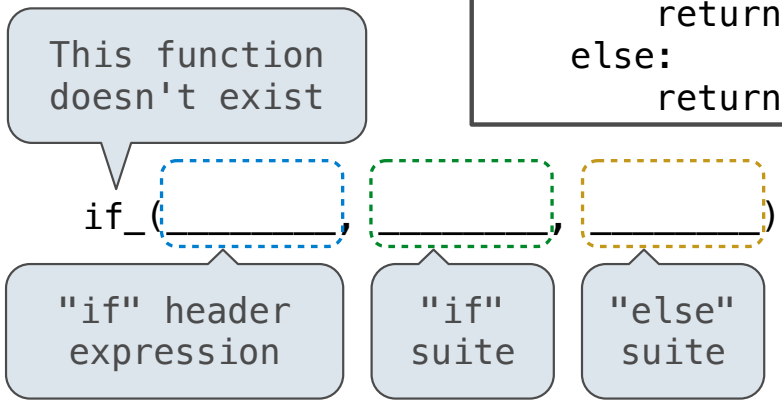

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2. If the result is a false value **v**, then the expression evaluates to **v**.
3. Otherwise, the expression evaluates to the value of the subexpression **<right>**.

To evaluate the expression **<left>** **or** **<right>**:

1. Evaluate the subexpression **<left>**.
2. If the result is a true value **v**, then the expression evaluates to **v**.
3. Otherwise, the expression evaluates to the value of the subexpression **<right>**.

Logical Operators

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1. Evaluate the subexpression **<left>**.
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(Demo)

Conditional Expressions

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<consequent> if <predicate> else <alternative>
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3. Otherwise, the value of the whole expression is the value of the **<alternative>**.

```
>>> x = 0
>>> abs(1/x if x != 0 else 0)
0
```