Environments

Announcements

Environments for Higher-Order Functions

Environments Enable Higher-Order Functions

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Environment diagrams describe how higher-order functions work!

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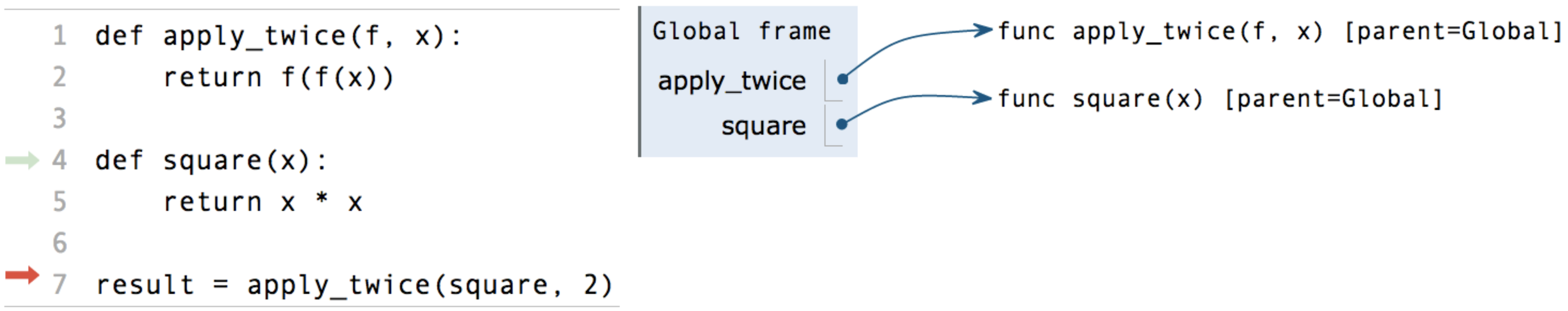
Environment diagrams describe how higher-order functions work!

(Demo)

Names can be Bound to Functional Arguments



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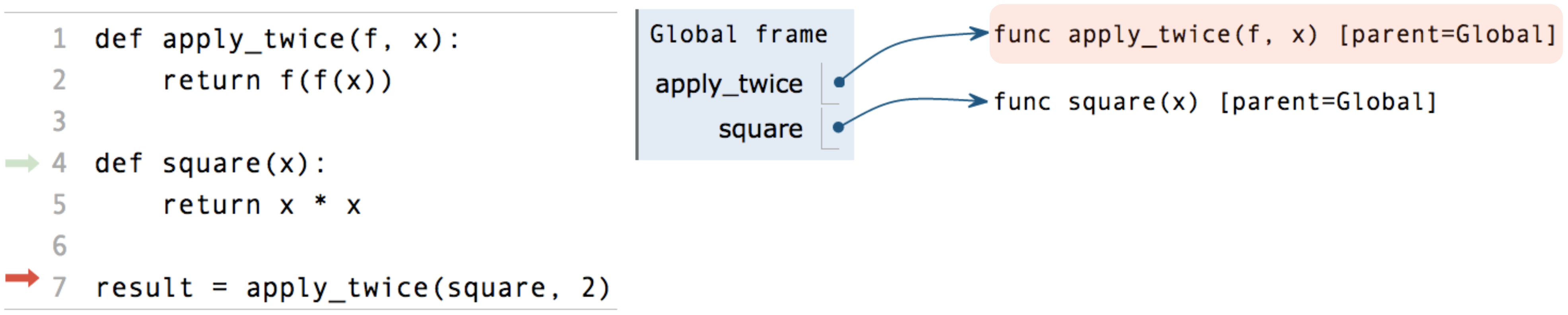

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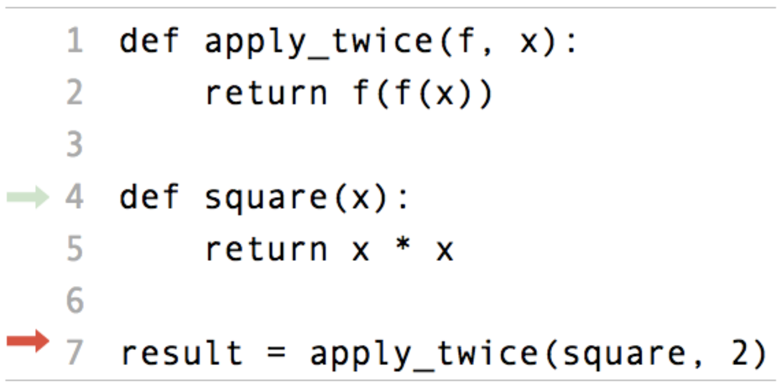

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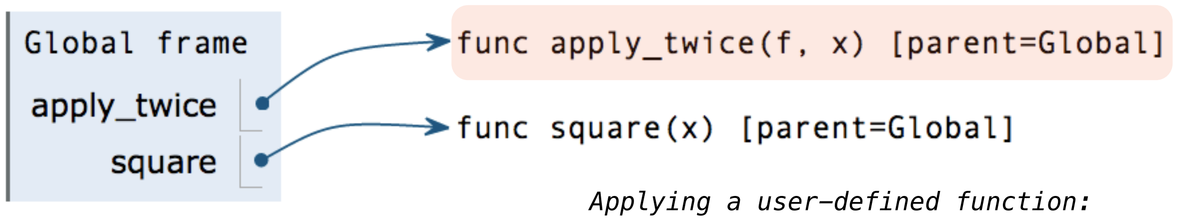
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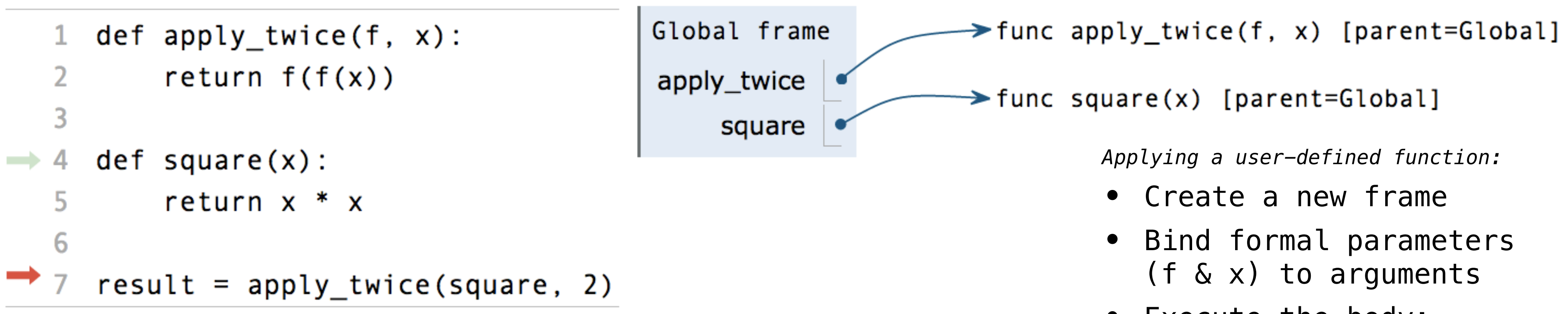

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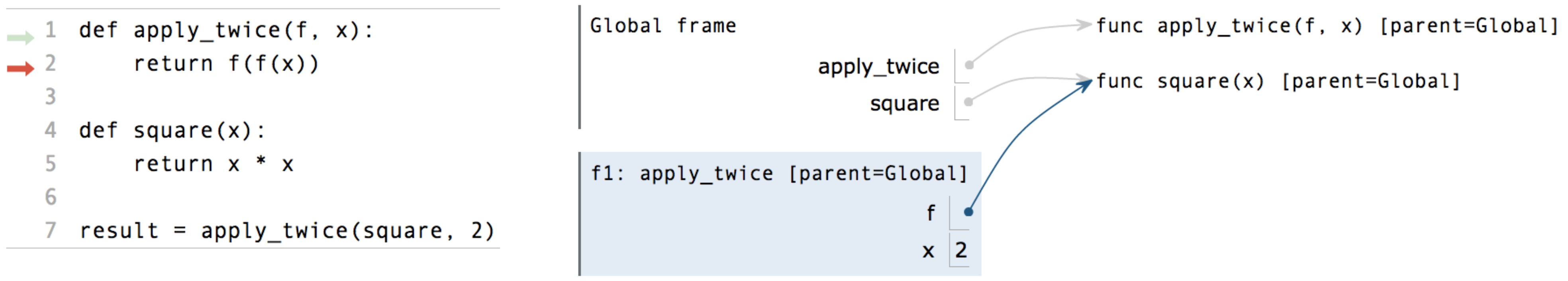

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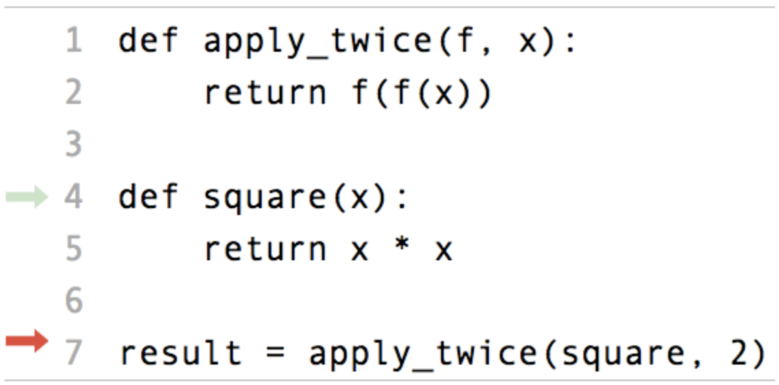

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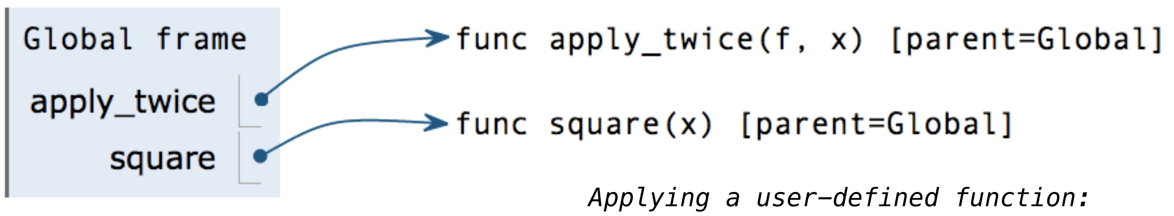

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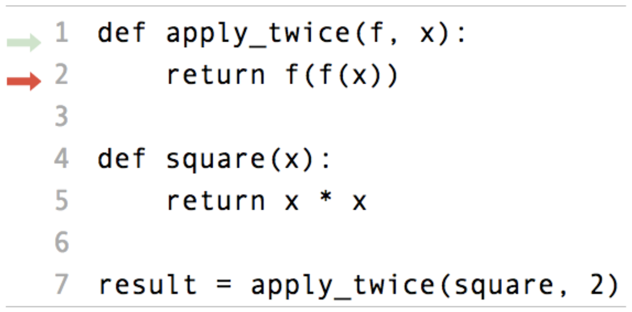

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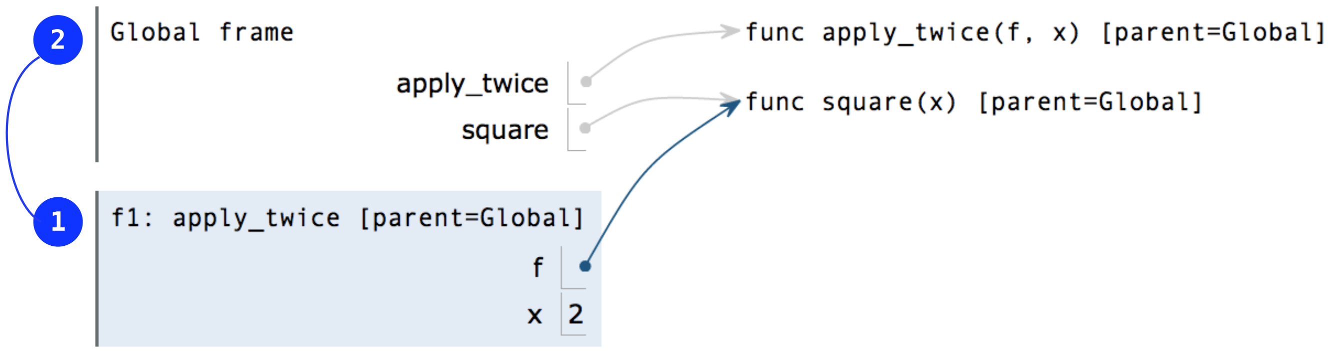

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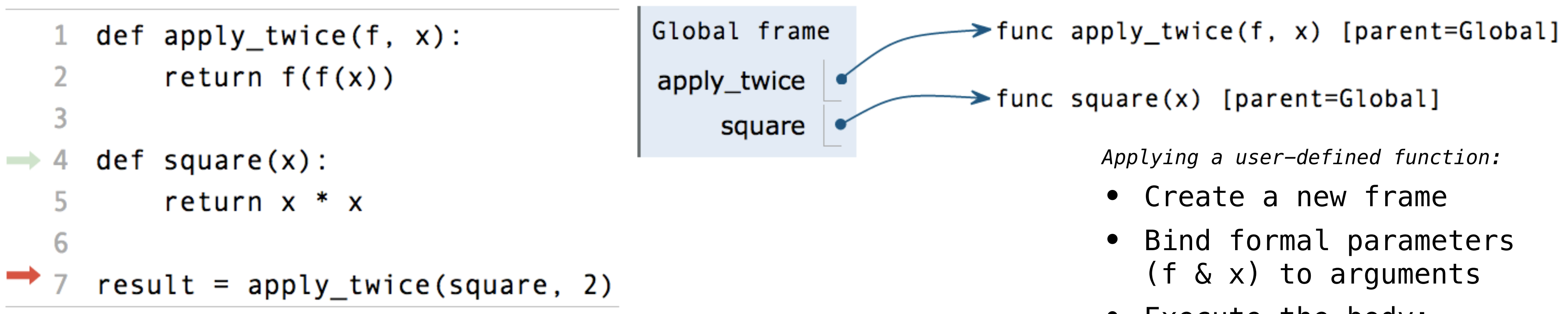

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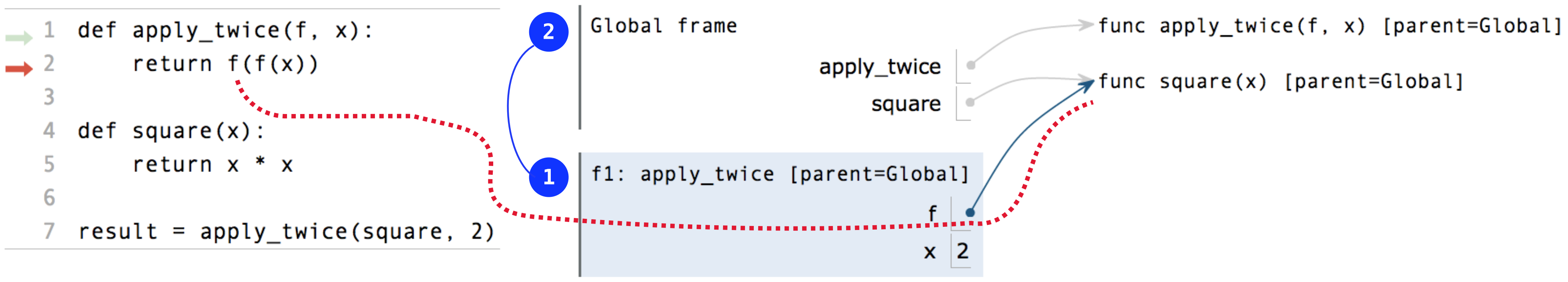

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Environments for Nested Definitions

(Demo)

Environment Diagrams for Nested Def Statements

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def make_adder(n):
    def adder(k):
        return k + n
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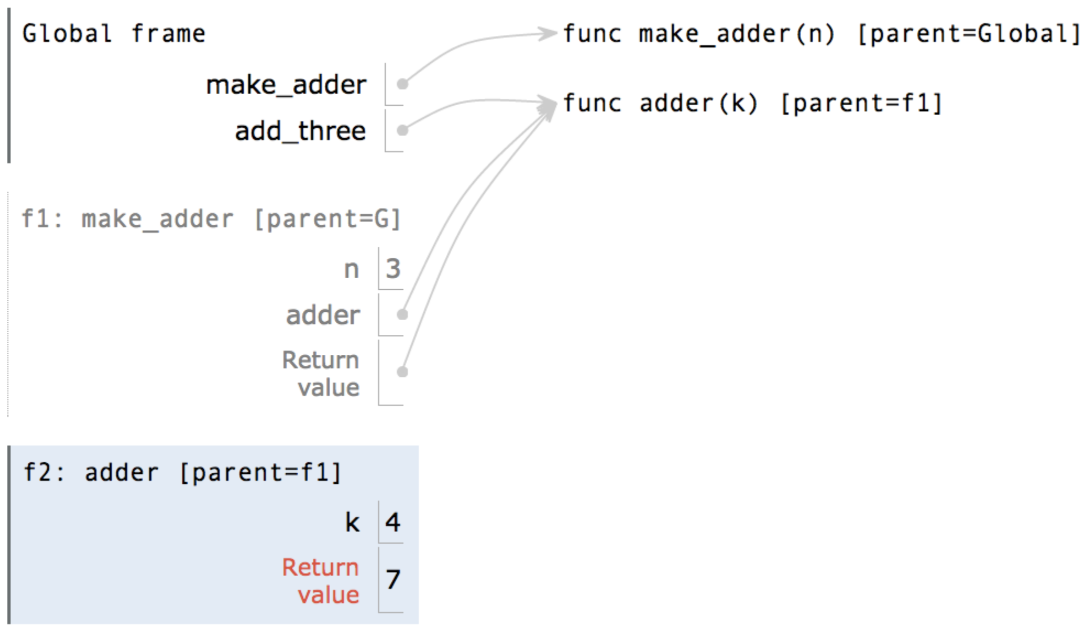
add_three = make_adder(3)
add_three(4)
```



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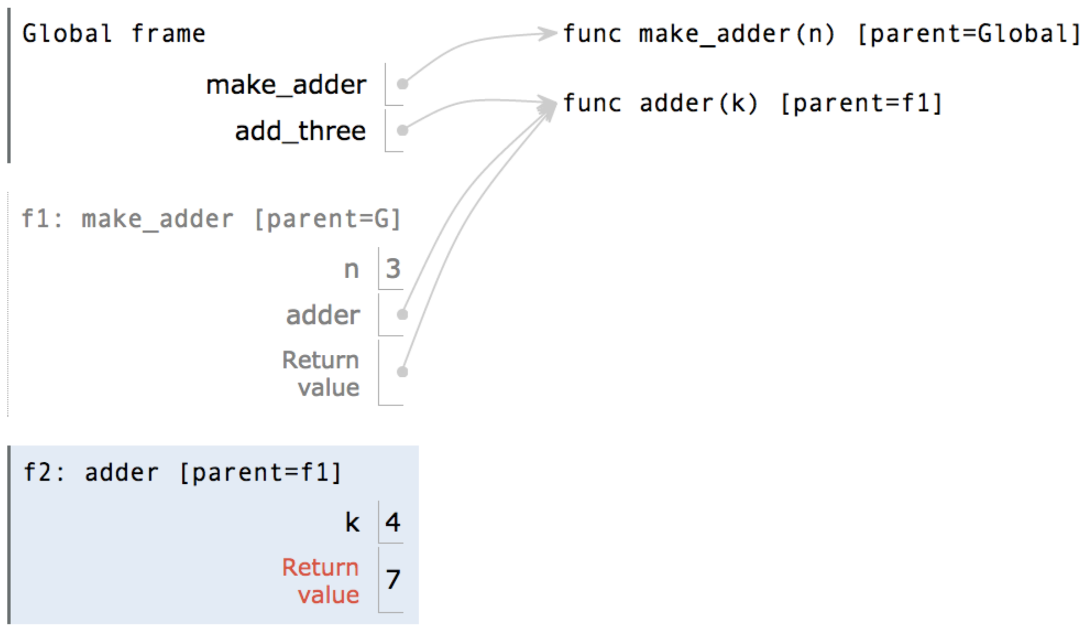

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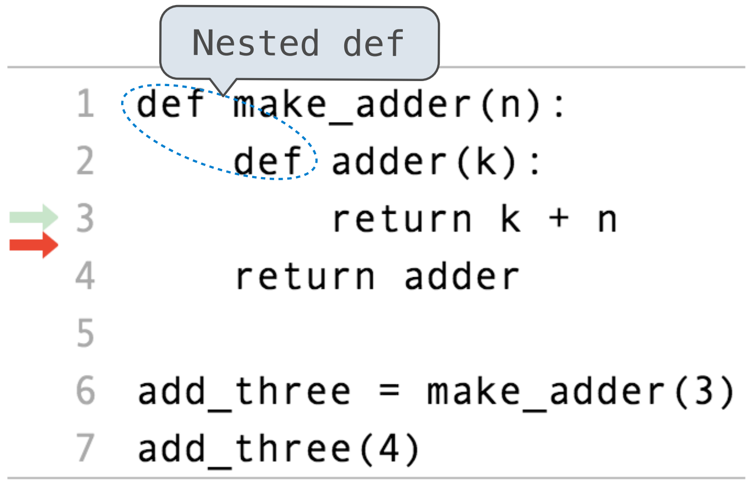

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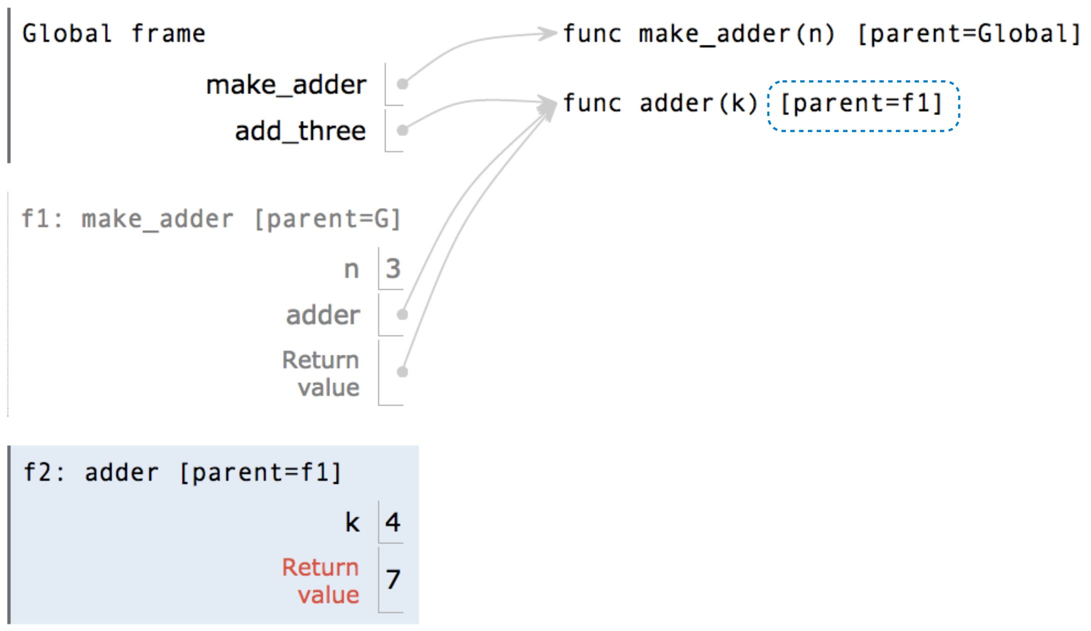

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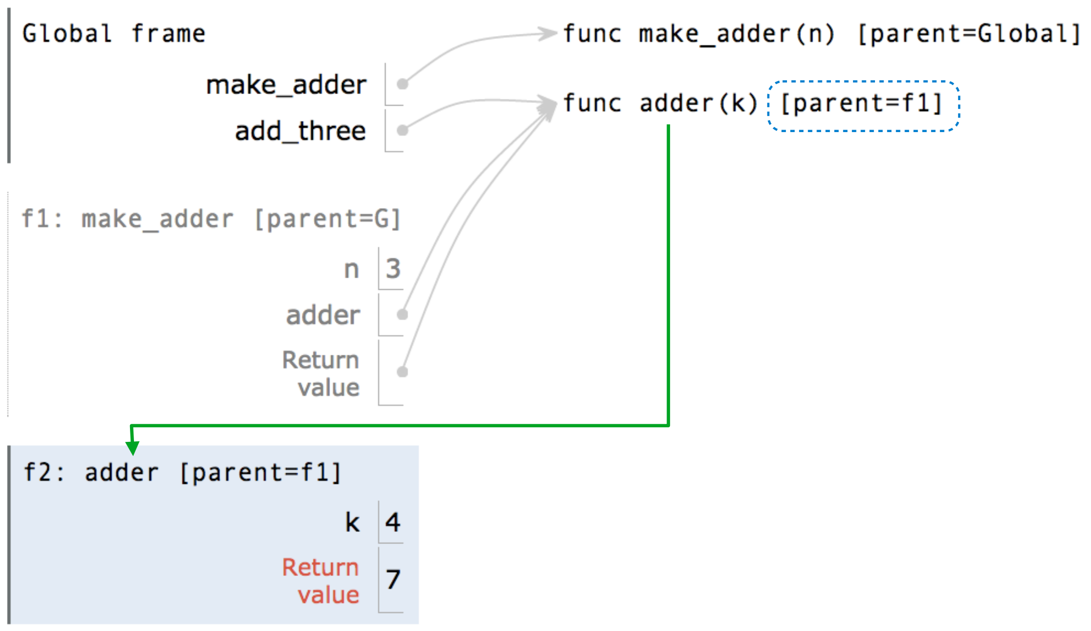

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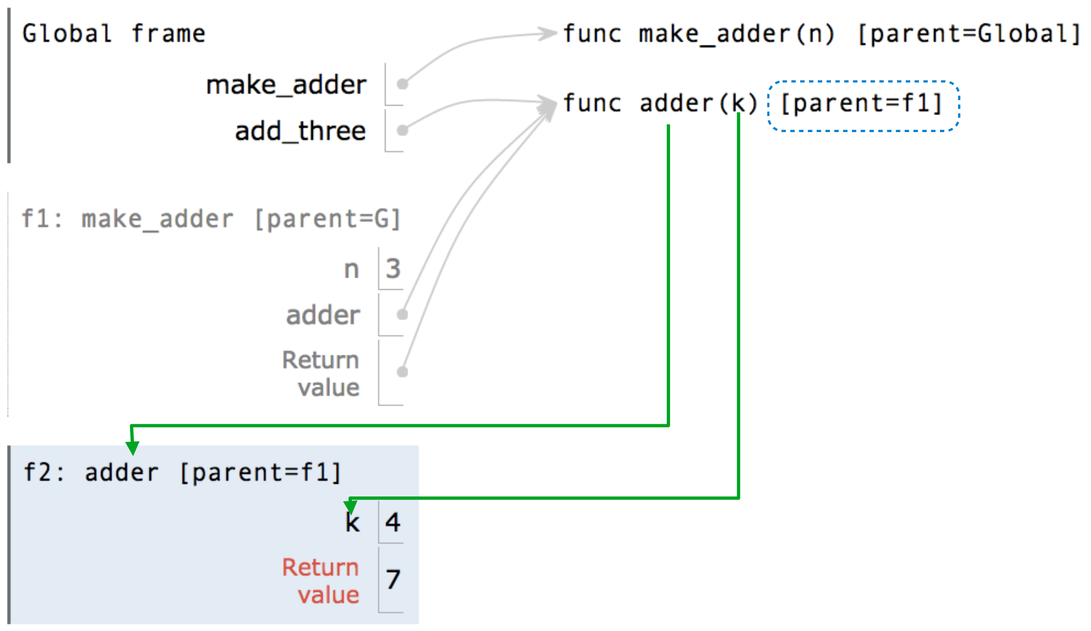

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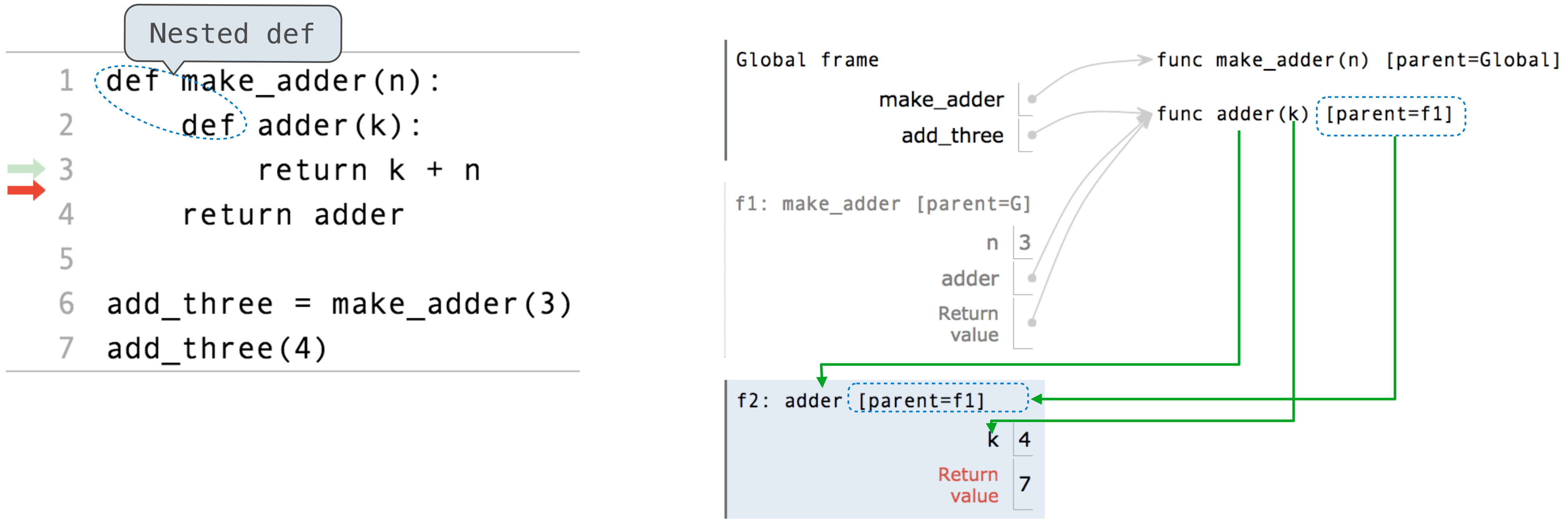

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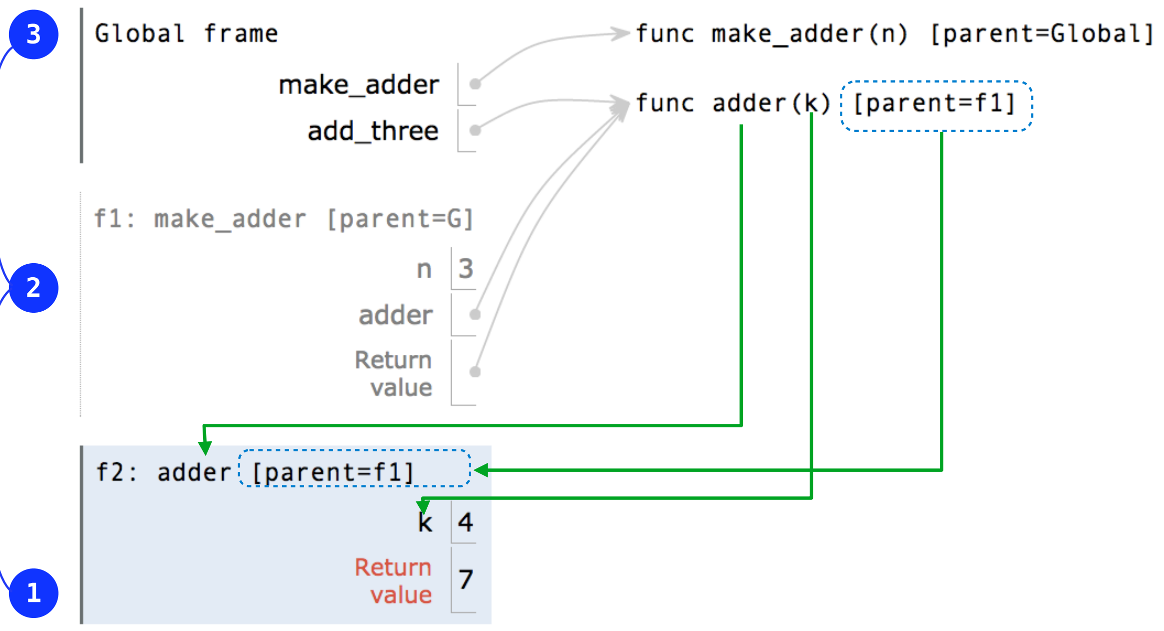

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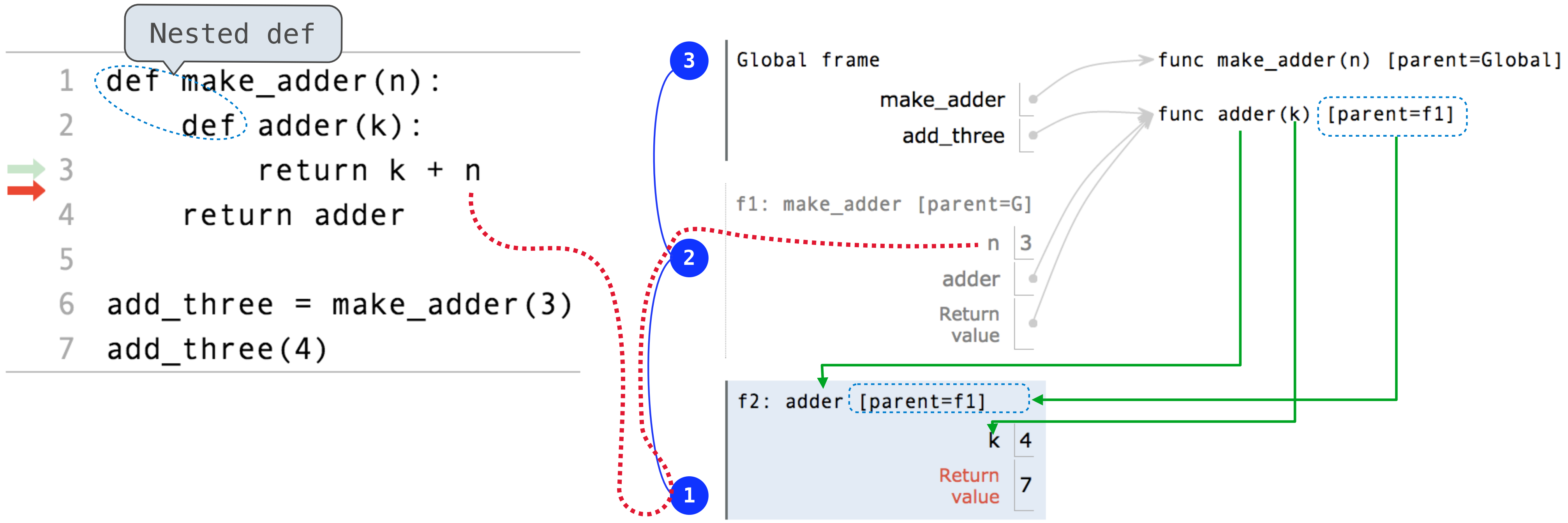

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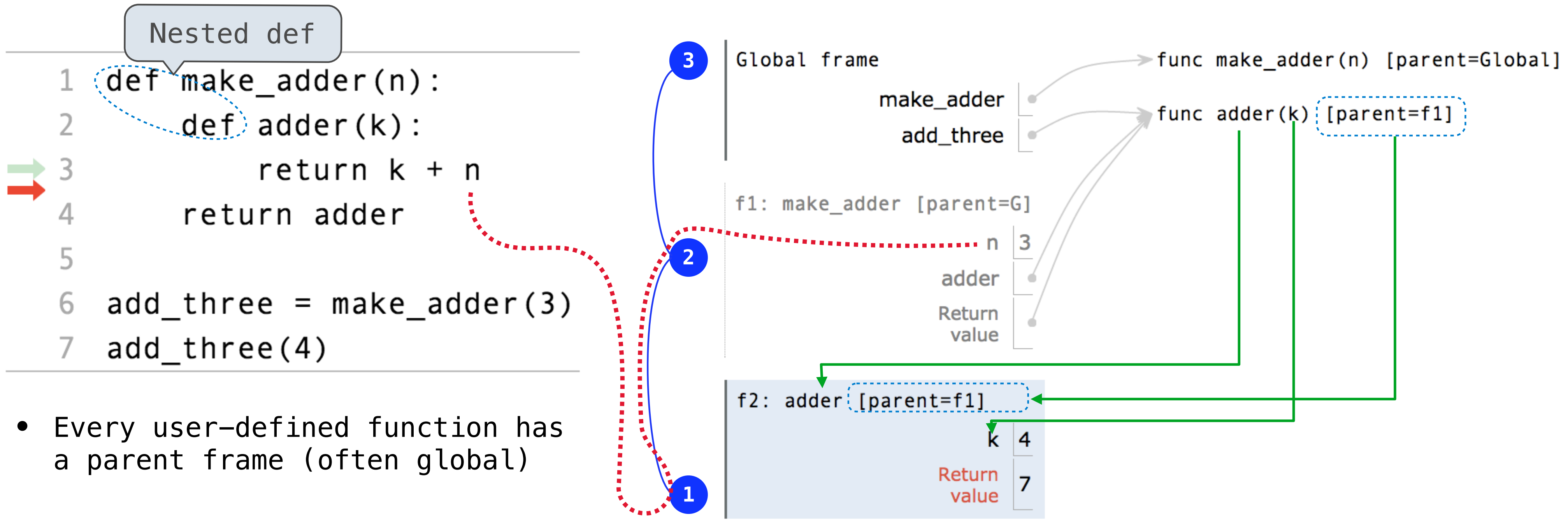

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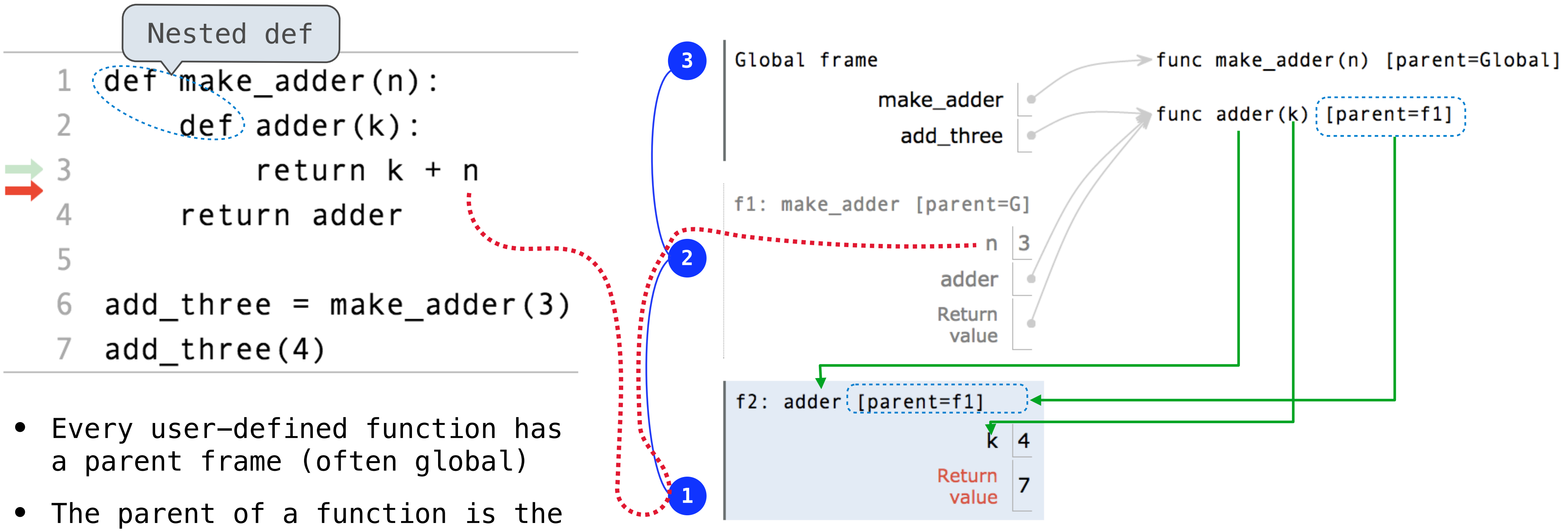

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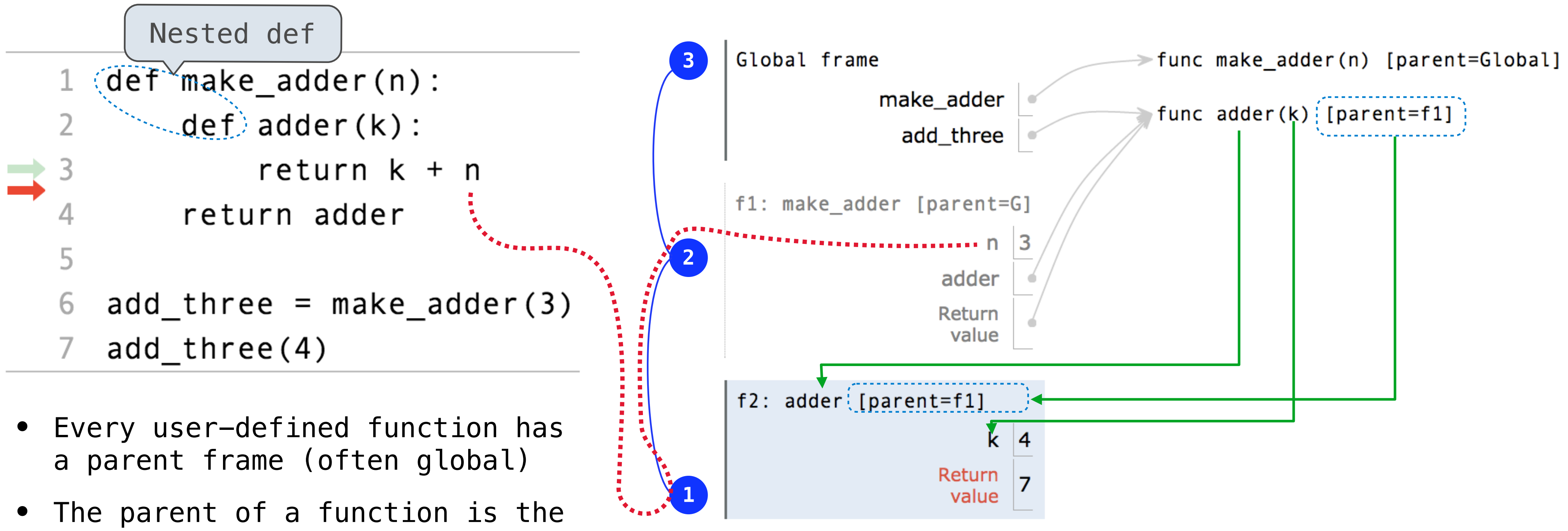

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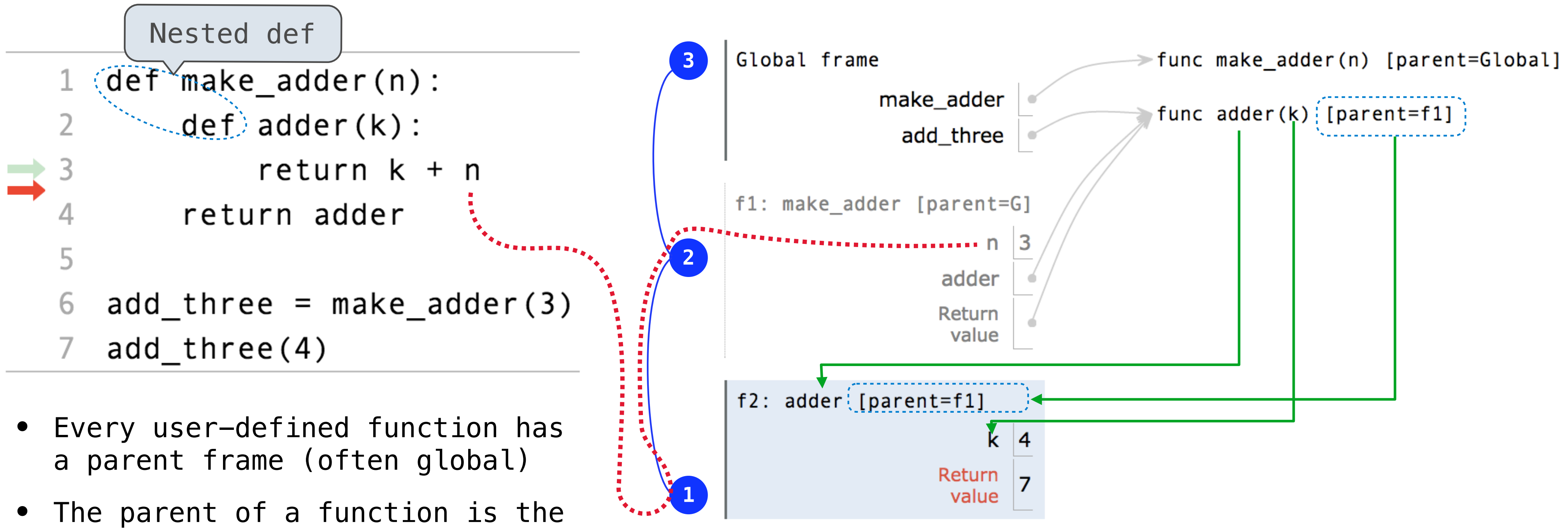

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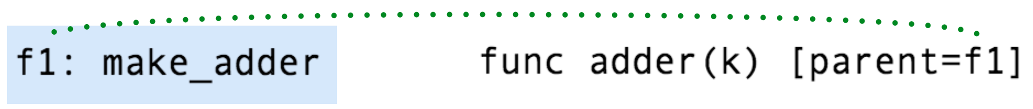

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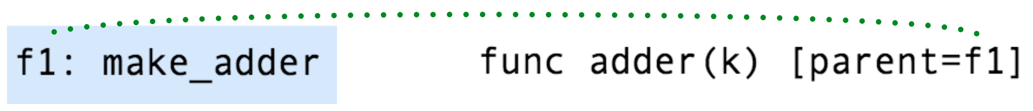

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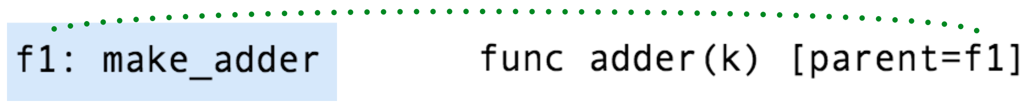

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3. Bind the <formal parameters> to the arguments in the local frame.

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```

Bind <name> to the function value in the current frame

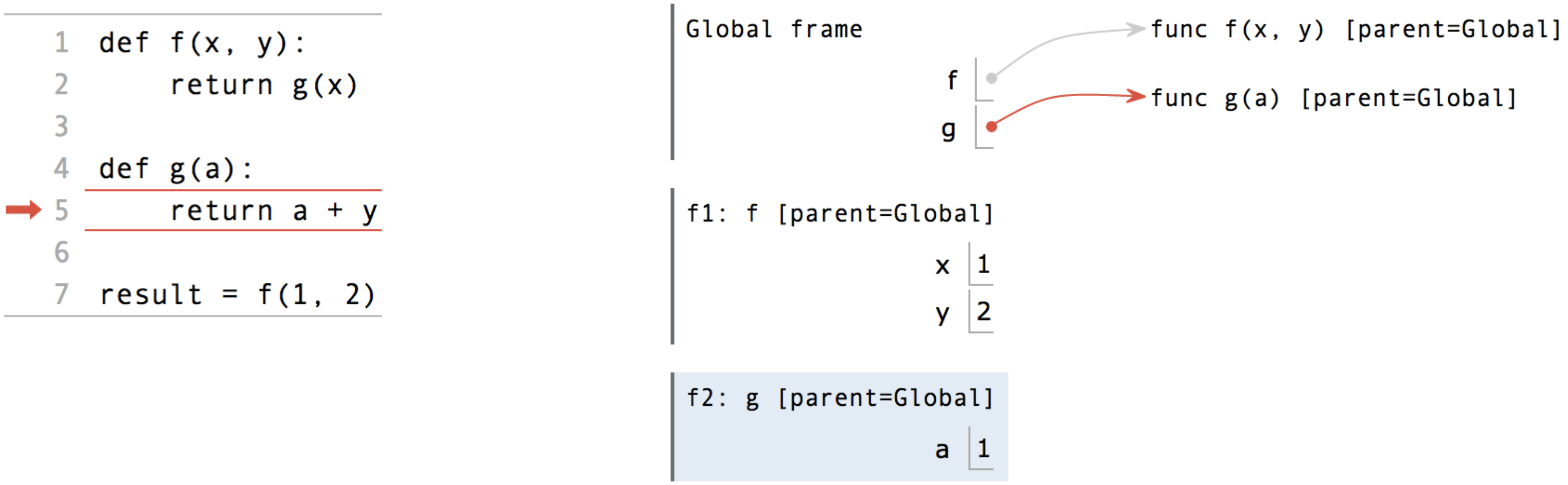
When a function is called:

1. Add a local frame, titled with the <name> of the function being called.
2. Copy the parent of the function to the local frame: [parent=<label>]
3. Bind the <formal parameters> to the arguments in the local frame.
4. Execute the body of the function in the environment that starts with the local frame.

Local Names

(Demo)

Local Names are not Visible to Other (Non-Nested) Functions



```
def f(x, y):
    return g(x)

def g(a):
    return a + y

result = f(1, 2)
```

http://pythontutor.com/composingprograms.html#code=def%20f%28x,
%20y%29%3A%0A%20%20%20%20return%20g%28x%29%0A%0Adef%20g%28a%29%3A%0A%20%20%20%20return%20a%20%2B%20y%0A%20%20%20%20%0Aresult%20%3D%20f%281,%202%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

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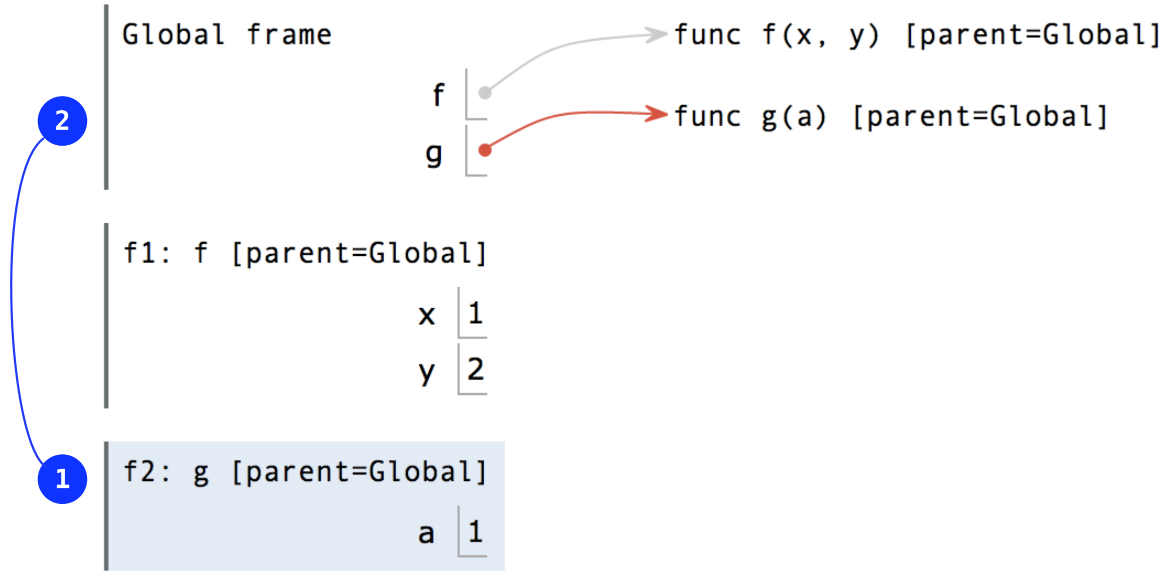

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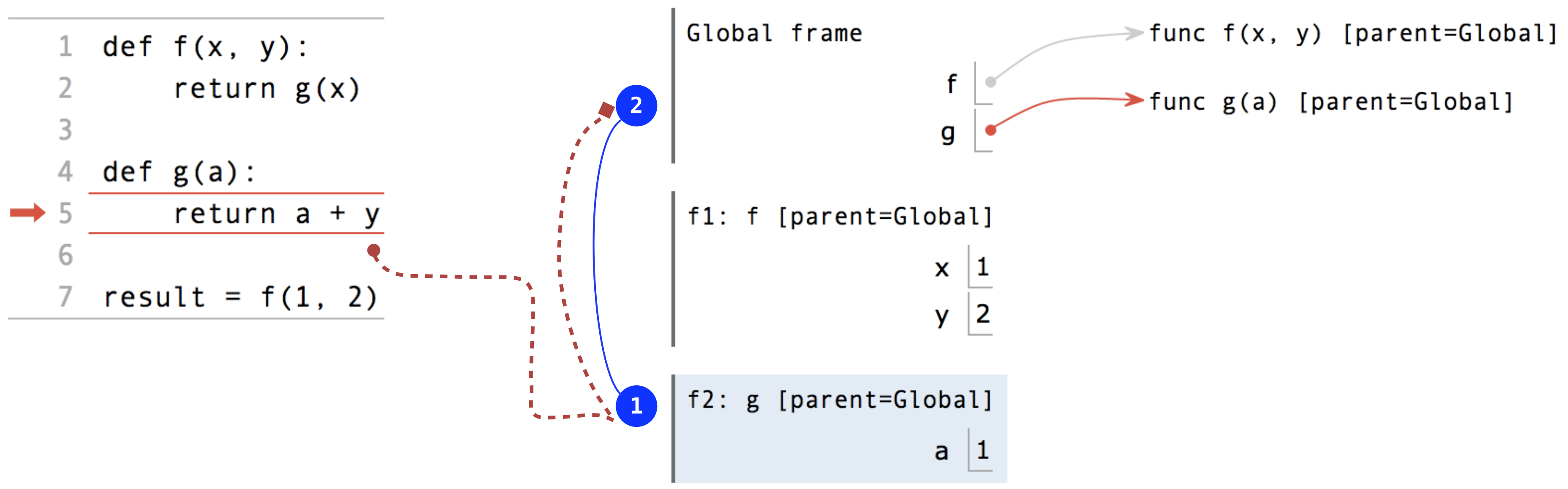

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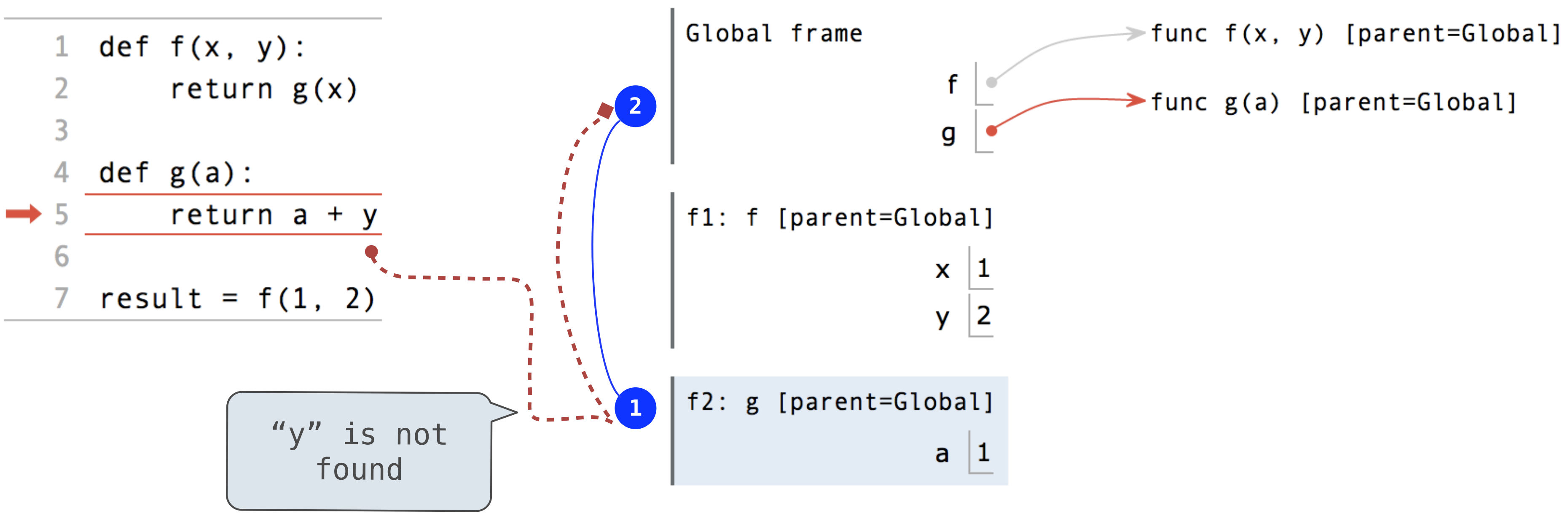

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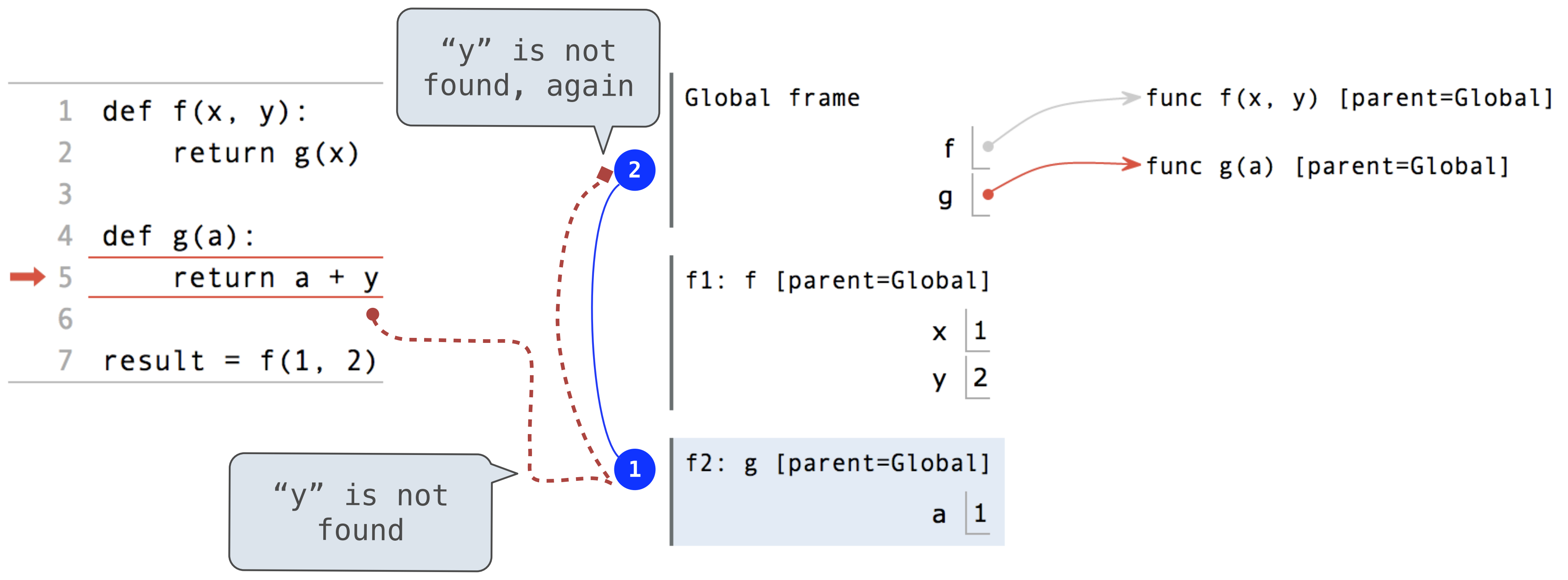

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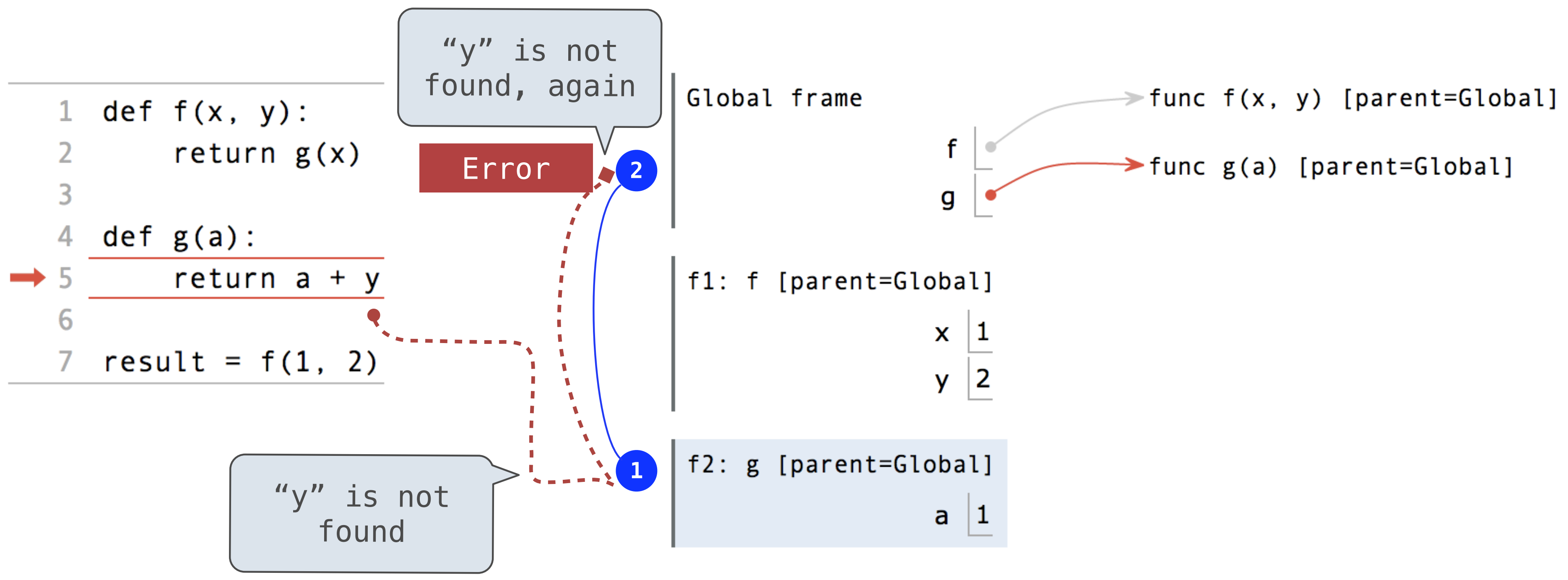

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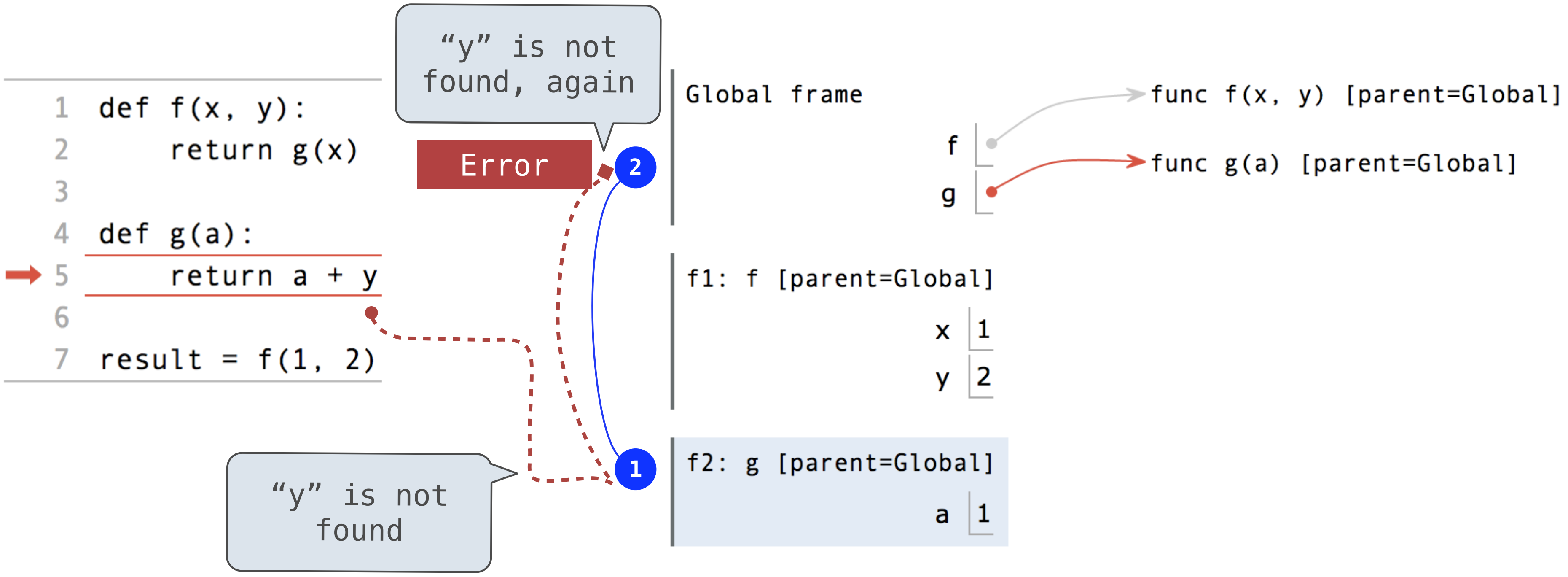

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```

- An environment is a sequence of frames.

http://pythontutor.com/composingprograms.html#code=def%20f%28x,
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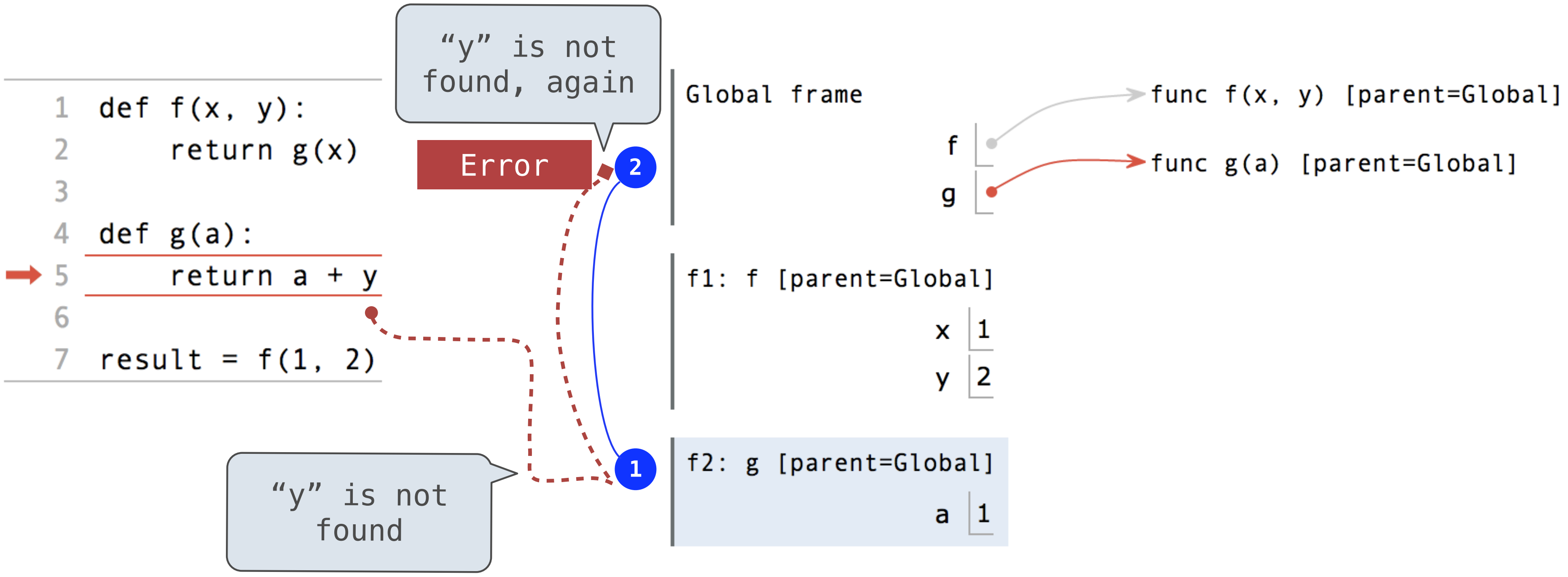

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result = f(1, 2)
```

- An environment is a sequence of frames.
- The environment created by calling a top-level function (no def within def) consists of one local frame, followed by the global frame.

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Lambda Expressions

(Demo)

Lambda Expressions

Lambda Expressions

```
>>> x = 10
```

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>>> square = x * x
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An expression: this one evaluates to a number

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A function

with formal parameter x

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Also an expression: evaluates to a function

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```

```
A function
    with formal parameter x
         that returns the value of "x * x"
```

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Must be a single expression

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>>> square(4)
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```

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Lambda expressions are not common in Python, but important in general

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Must be a single expression

```
>>> square(4)
16
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Lambda expressions are not common in Python, but important in general

Lambda expressions in Python cannot contain statements at all!

Lambda Expressions Versus Def Statements

Lambda Expressions Versus Def Statements

vs

Lambda Expressions Versus Def Statements

```
square = lambda x: x * x
```

VS

Lambda Expressions Versus Def Statements

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square = lambda x: x * x
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VS

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def square(x):
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Lambda Expressions Versus Def Statements

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def square(x):
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- Both create a function with the same domain, range, and behavior.

Lambda Expressions Versus Def Statements

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square = lambda x: x * x
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VS

```
def square(x):
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- Both create a function with the same domain, range, and behavior.
- Both bind that function to the name square.

Lambda Expressions Versus Def Statements

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square = lambda x: x * x
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```
def square(x):
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- Both create a function with the same domain, range, and behavior.
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- Only the def statement gives the function an intrinsic name, which shows up in environment diagrams but doesn't affect execution (unless the function is printed).

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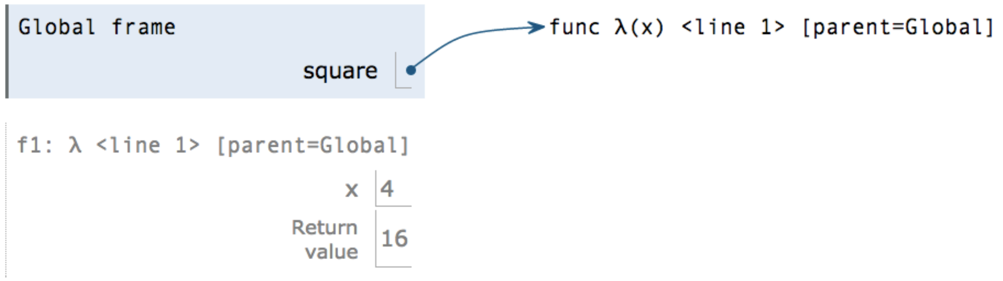

```
Global frame
                square  •──> func λ(x) <line 1> [parent=Global]

f1: λ <line 1> [parent=Global]
                     x  4
          Return value  16
```

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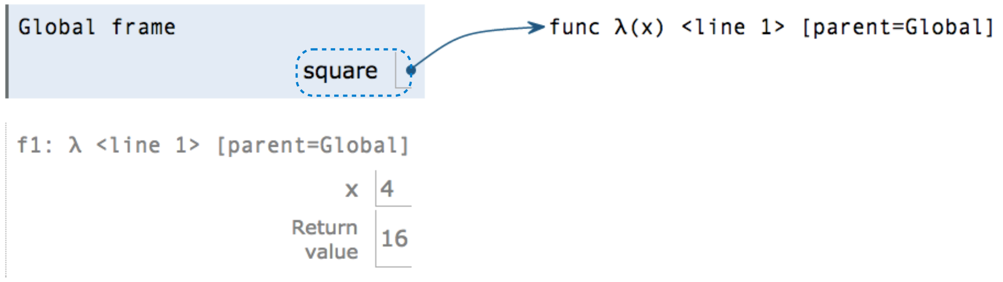

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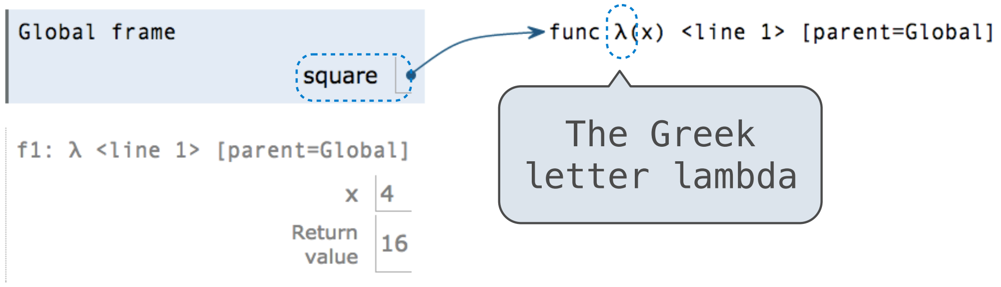
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The Greek letter lambda

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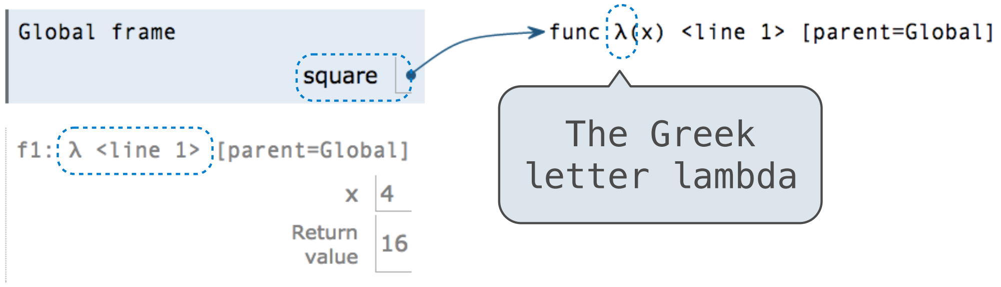
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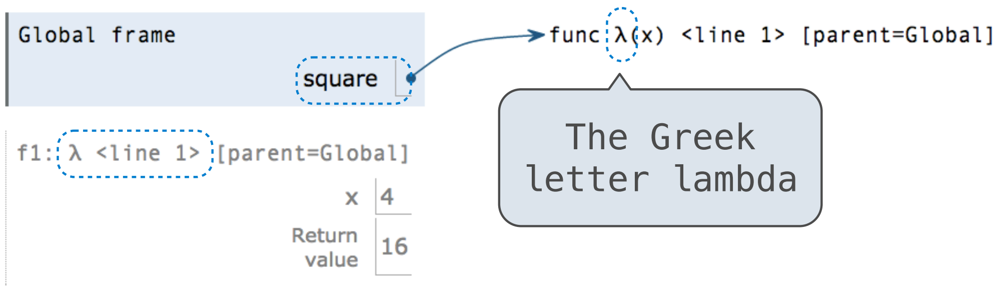

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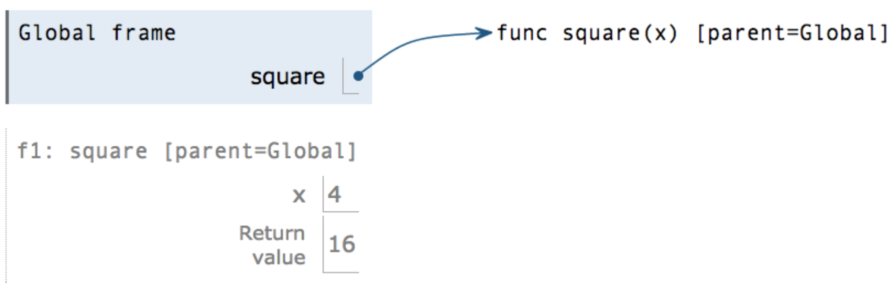


Function Composition

(Demo)

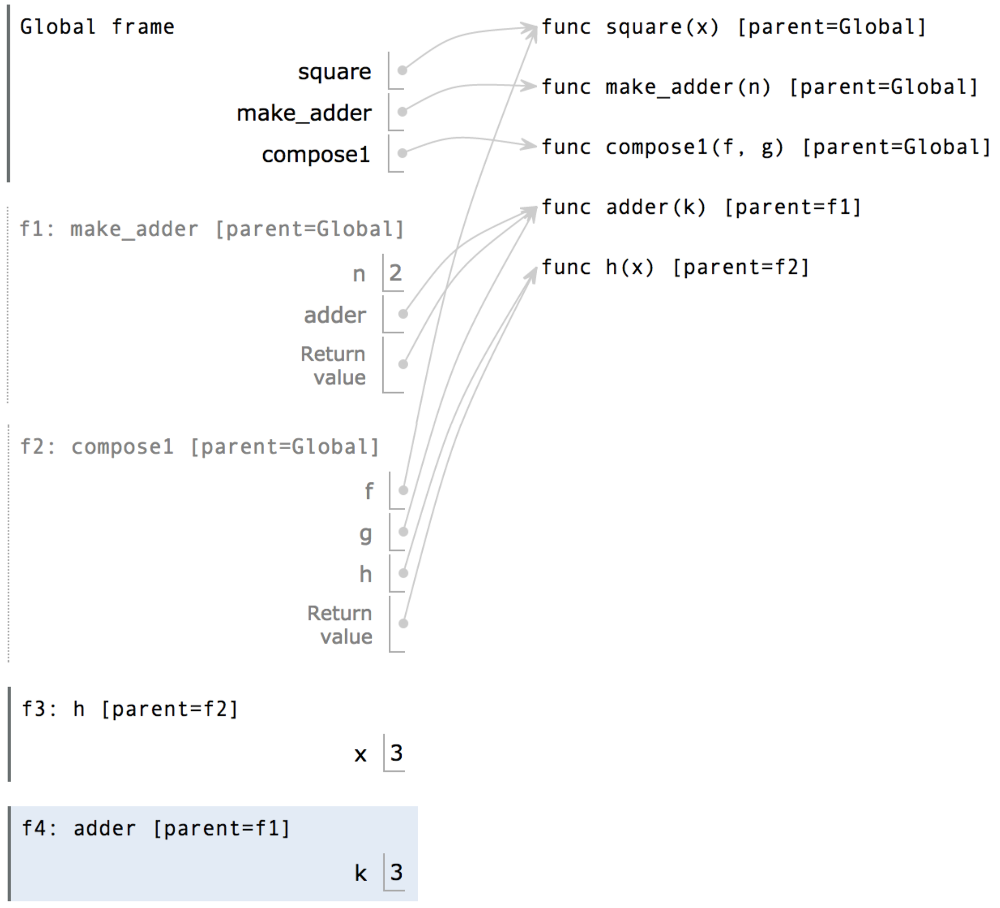
The Environment Diagram for Function Composition

```python
def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return k + n
    return adder

def compose1(f, g):
    def h(x):
        return f(g(x))
    return h

compose1(square, make_adder(2))(3)
```



Global frame
square
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func square(x) [parent=Global]
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func adder(k) [parent=f1]
func h(x) [parent=f2]
f1: make_adder [parent=Global]
n 2
adder
Return value
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f
g
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Return value
f3: h [parent=f2]
x 3
f4: adder [parent=f1]
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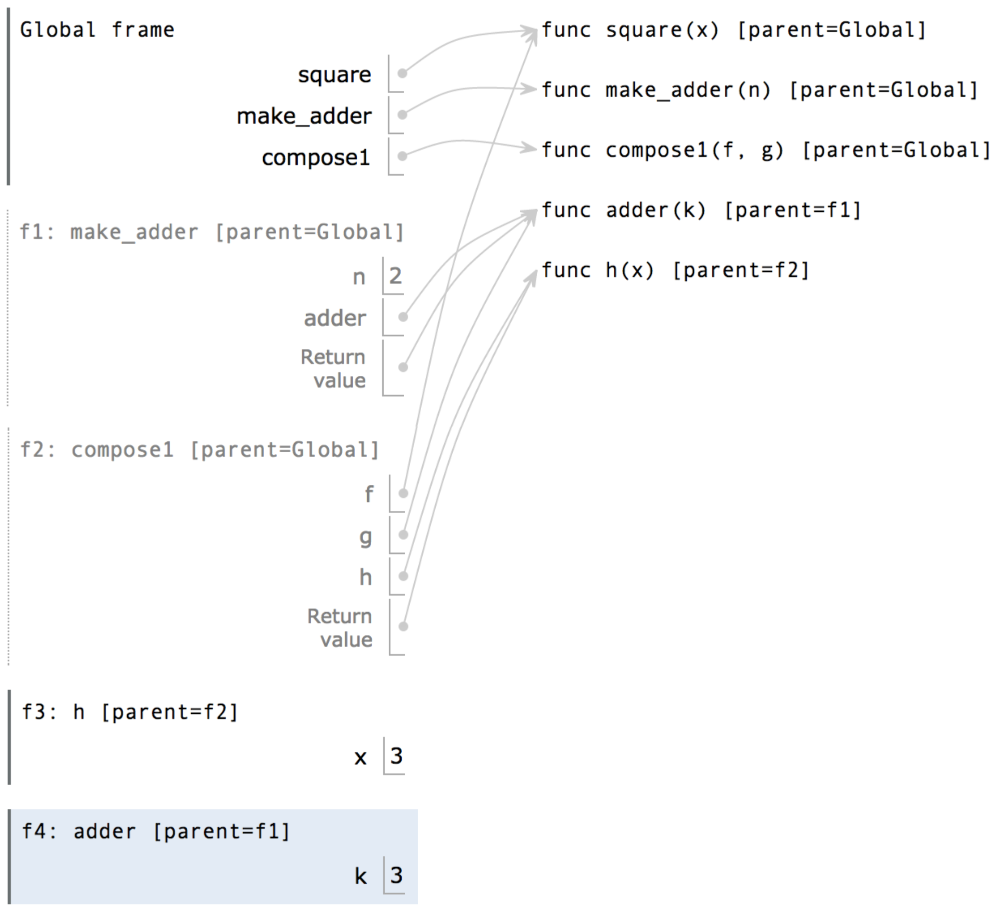

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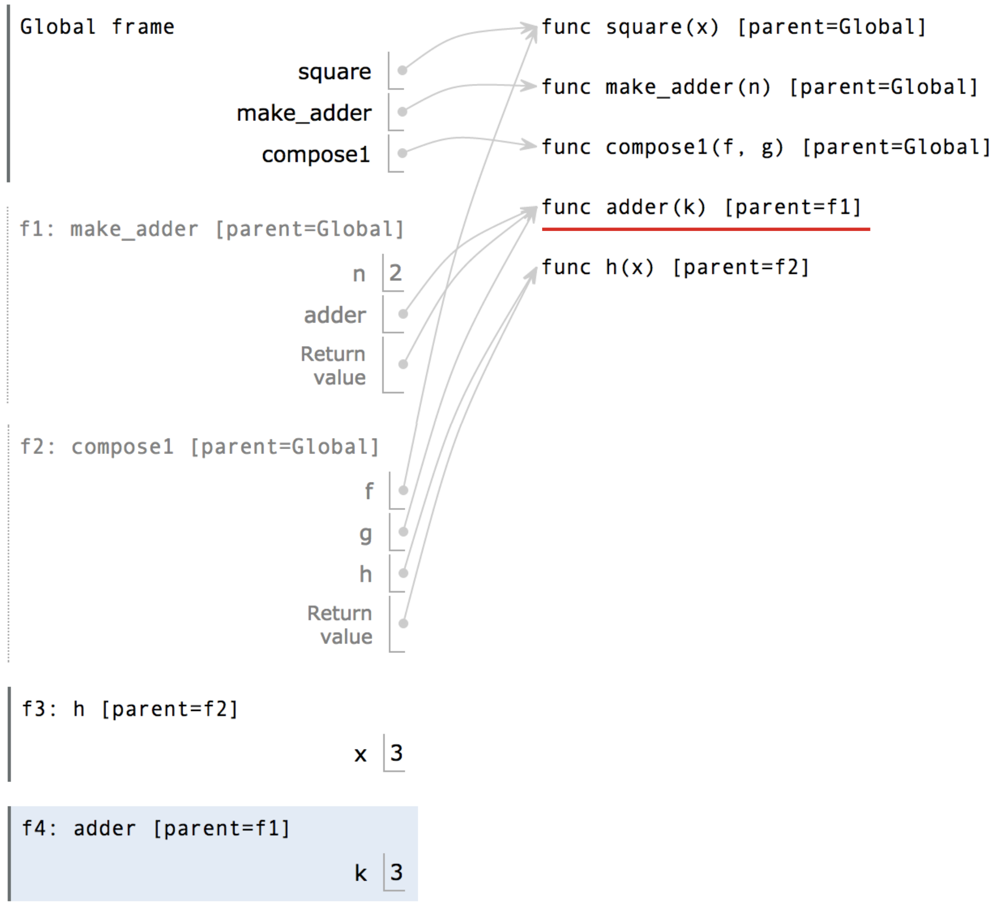
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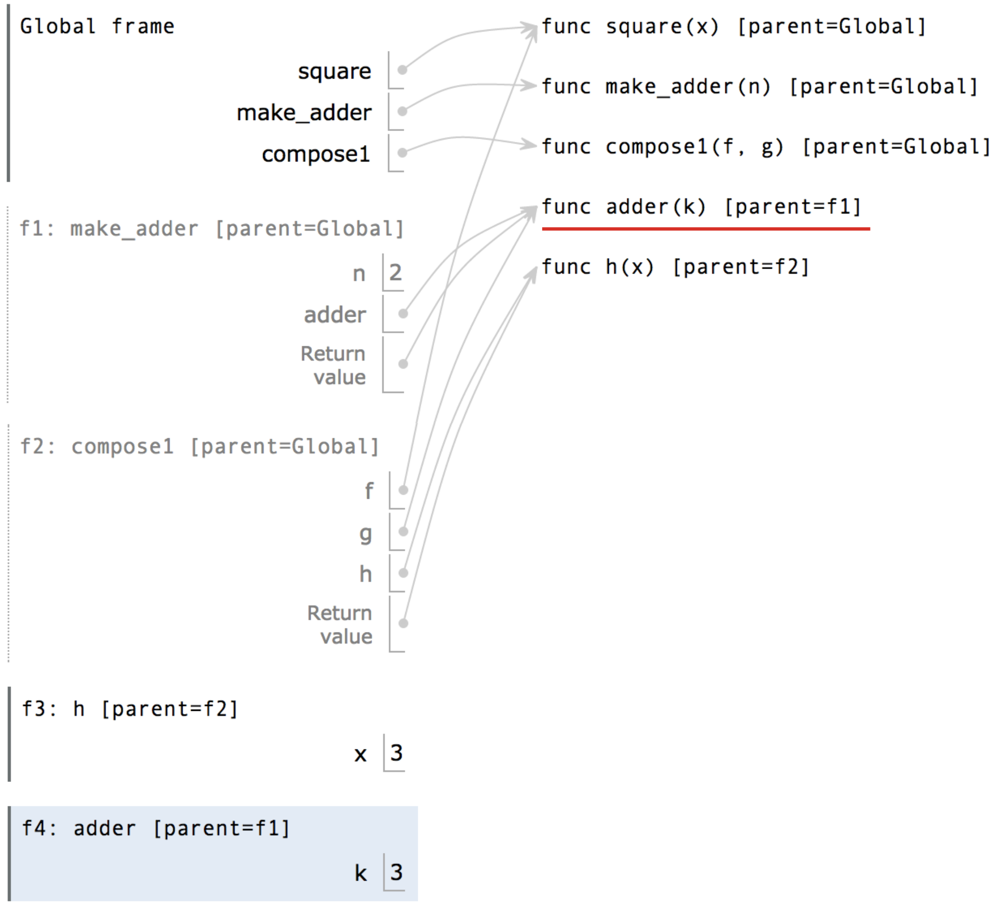

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f
g
h
Return value
f3: h [parent=f2]
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f4: adder [parent=f1]
k 3

http://pythontutor.com/composingprograms.html#code=def%20square%28x%29%3A%0A%20%20%20%20return%20x%20*%20x%0A%0Adef%20make_adder%28n%29%3A%0A%20%20%20%20def%20adder%28k%29%3A%0A%20%20%20%20%20%20%20%20return%20k%20%2B%20n%0A%20%20%20%20return%20adder%0A%20%20%20%20%0Adef%20compose1%28f,
%20g%29%3A%0A%20%20%20%20def%20h%28x%29%3A%0A%20%20%20%20%20%20%20%20return%20f%28g%28x%29%29%0A%20%20%20%20return%20h%0A%20%20%20%20%0Acompose1%28square,%20make_adder%282%29%29%283%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

The Environment Diagram for Function Composition

```python
def square(x):
    return x * x

def make_adder(n):
    def adder(k):
        return k + n
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def compose1(f, g):
    def h(x):
        return f(g(x))
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compose1(square, make_adder(2))(3)
```
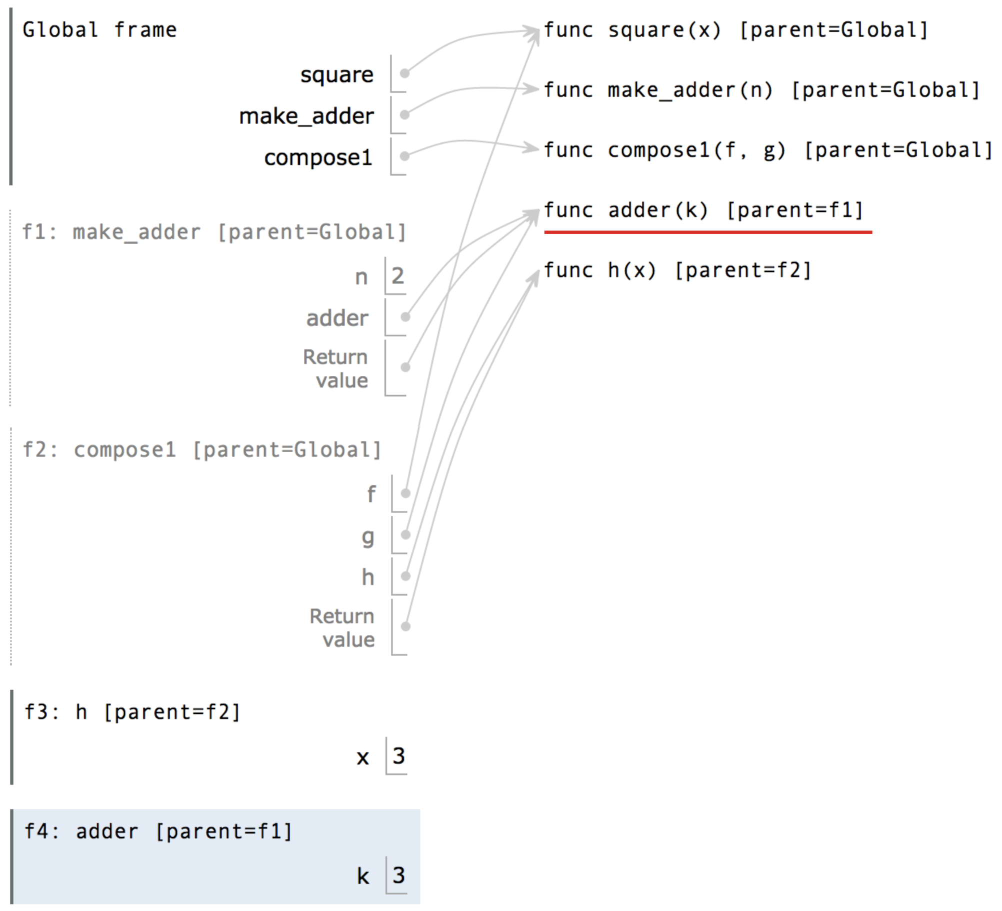

Return value of make_adder is an argument to compose1



Global frame
square
make_adder
compose1
f1: make_adder [parent=Global]
n 2
adder
Return value
f2: compose1 [parent=Global]
f
g
h
Return value
f3: h [parent=f2]
x 3
f4: adder [parent=f1]
k 3
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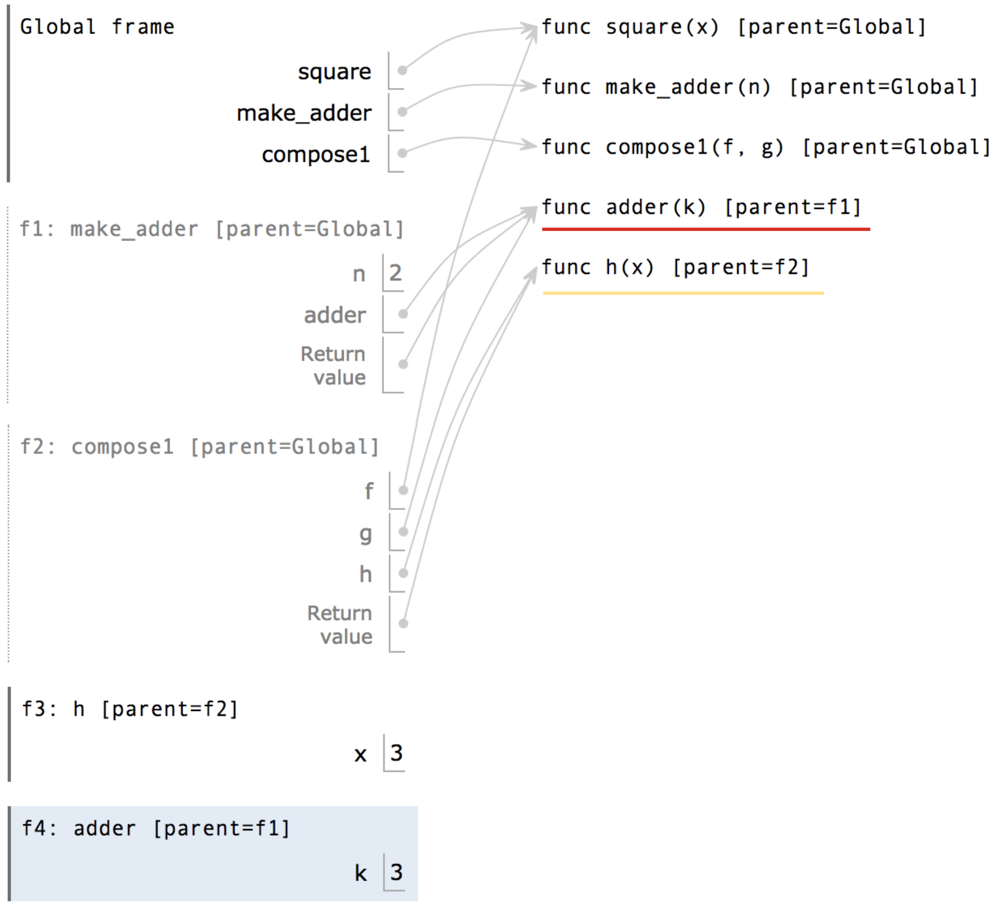

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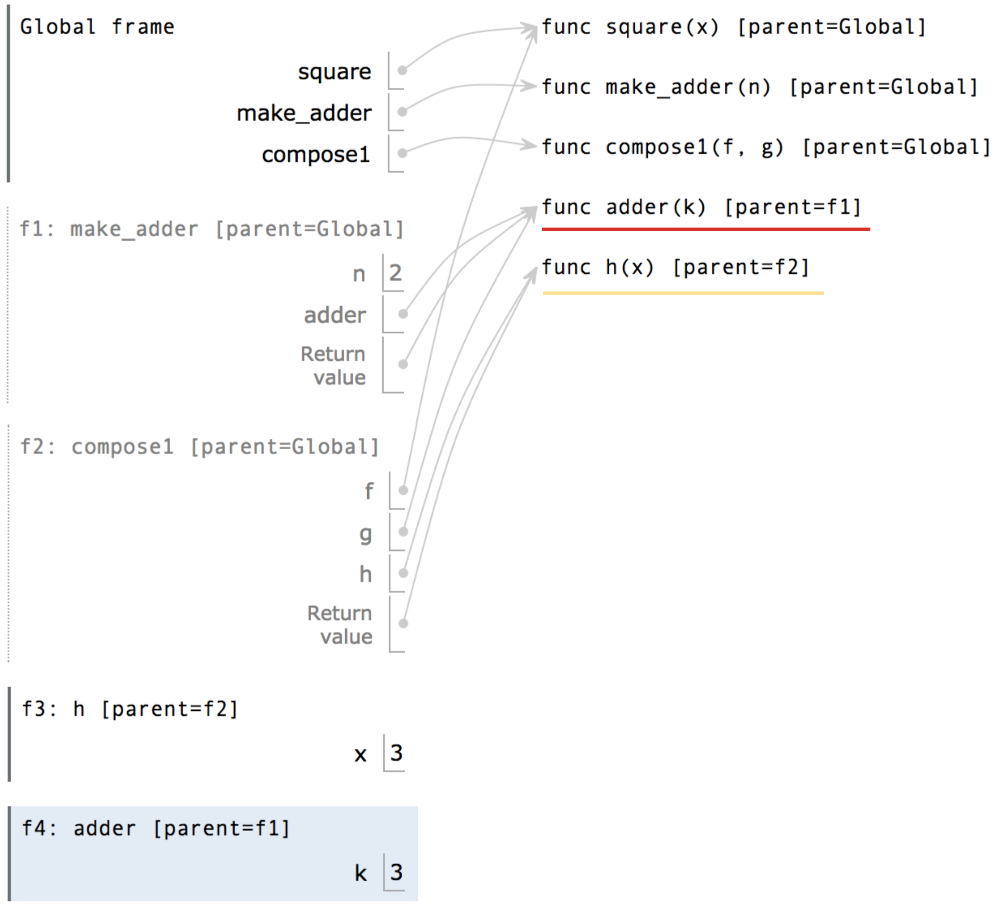

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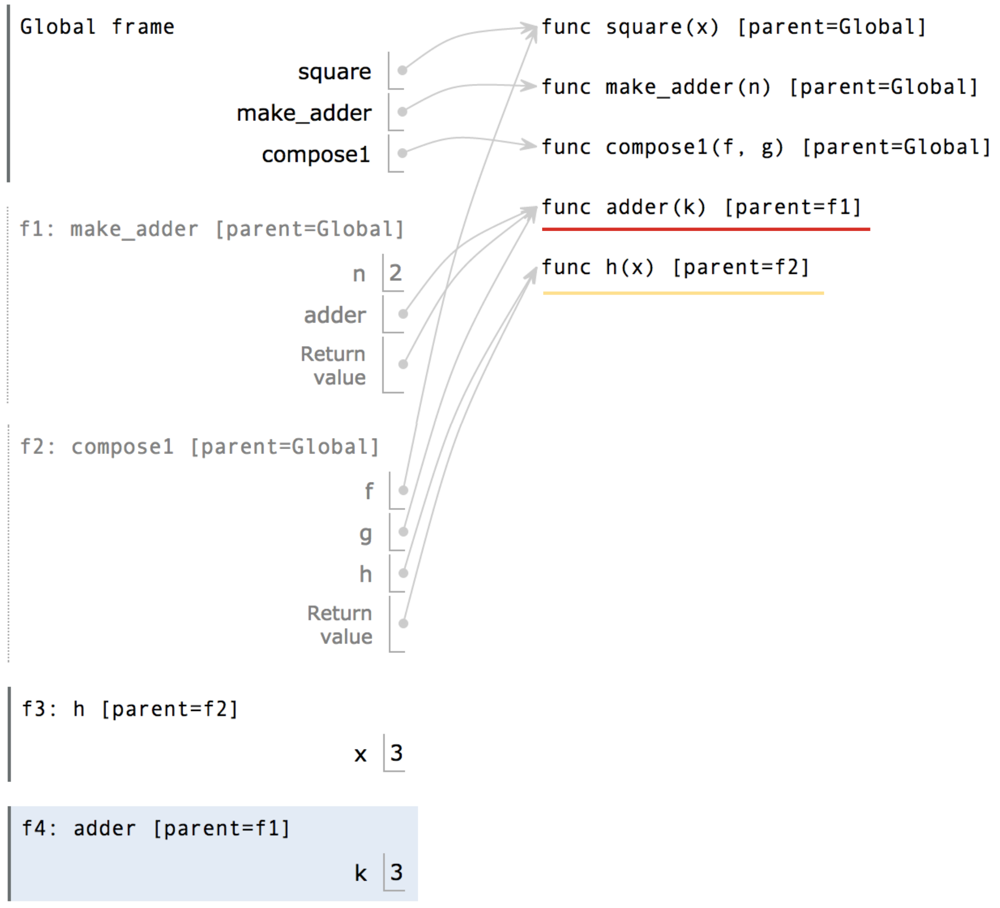

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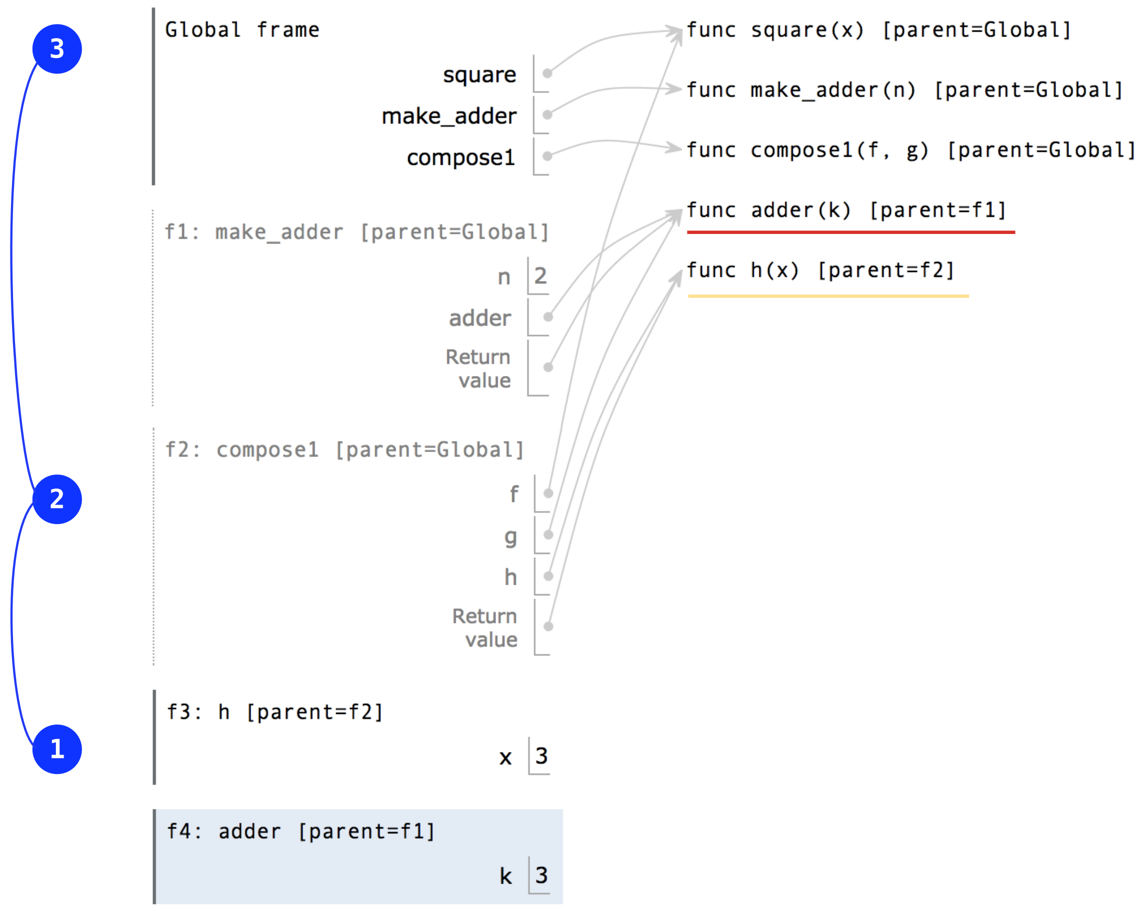

3
2
1
Global frame
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compose1
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adder
Return value
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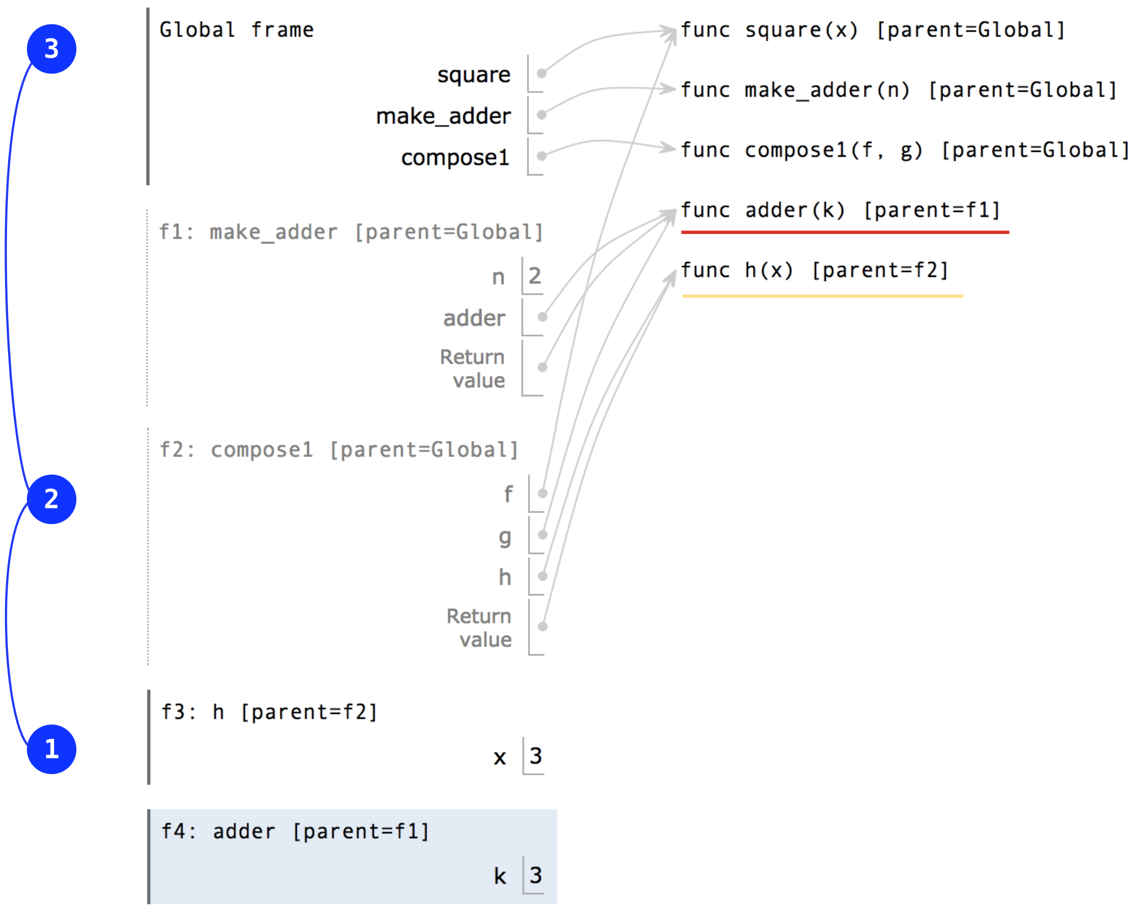

3
2
1
Global frame
square
make_adder
compose1
func square(x) [parent=Global]
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n 2
adder
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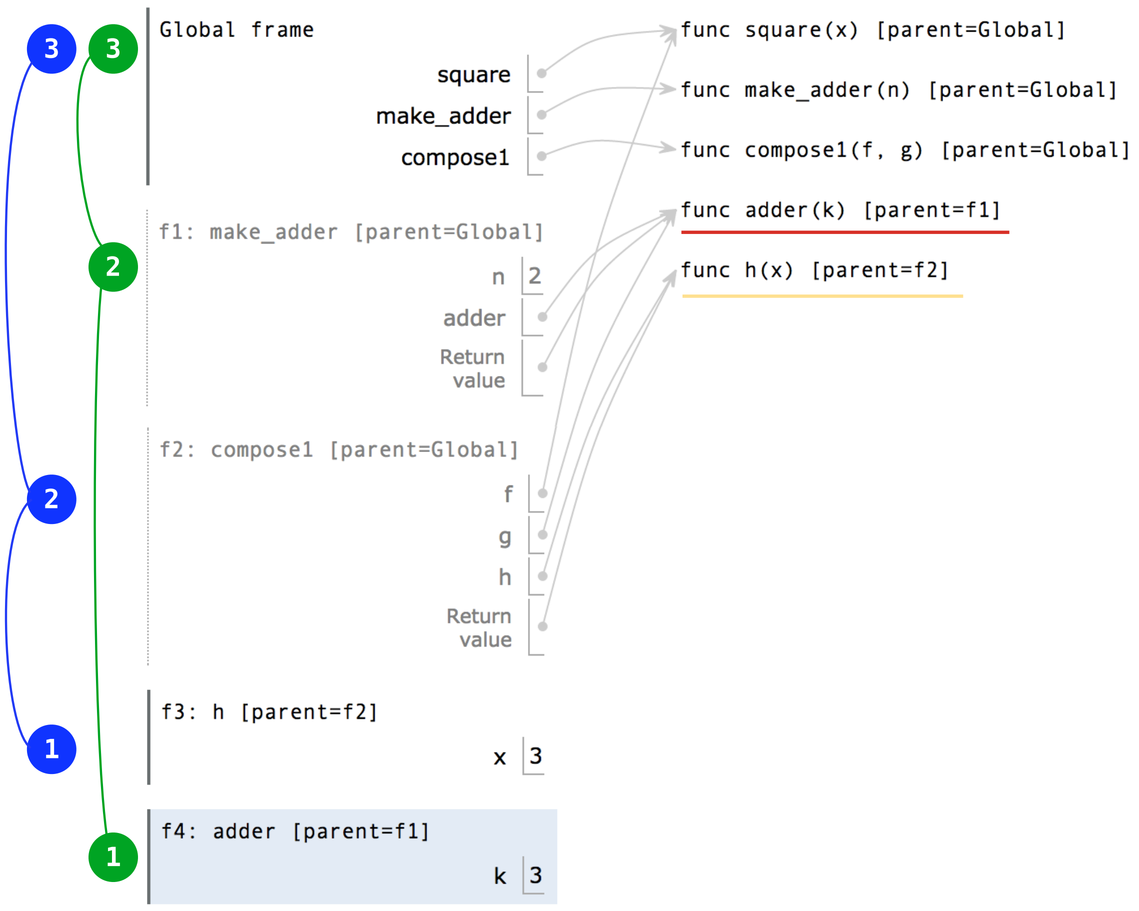


3
3
Global frame
square
make_adder
compose1
func square(x) [parent=Global]
func make_adder(n) [parent=Global]
func compose1(f, g) [parent=Global]
func adder(k) [parent=f1]
func h(x) [parent=f2]
2
f1: make_adder [parent=Global]
n 2
adder
Return value
2
f2: compose1 [parent=Global]
f
g
h
Return value
1
f3: h [parent=f2]
x 3
1
f4: adder [parent=f1]
k 3

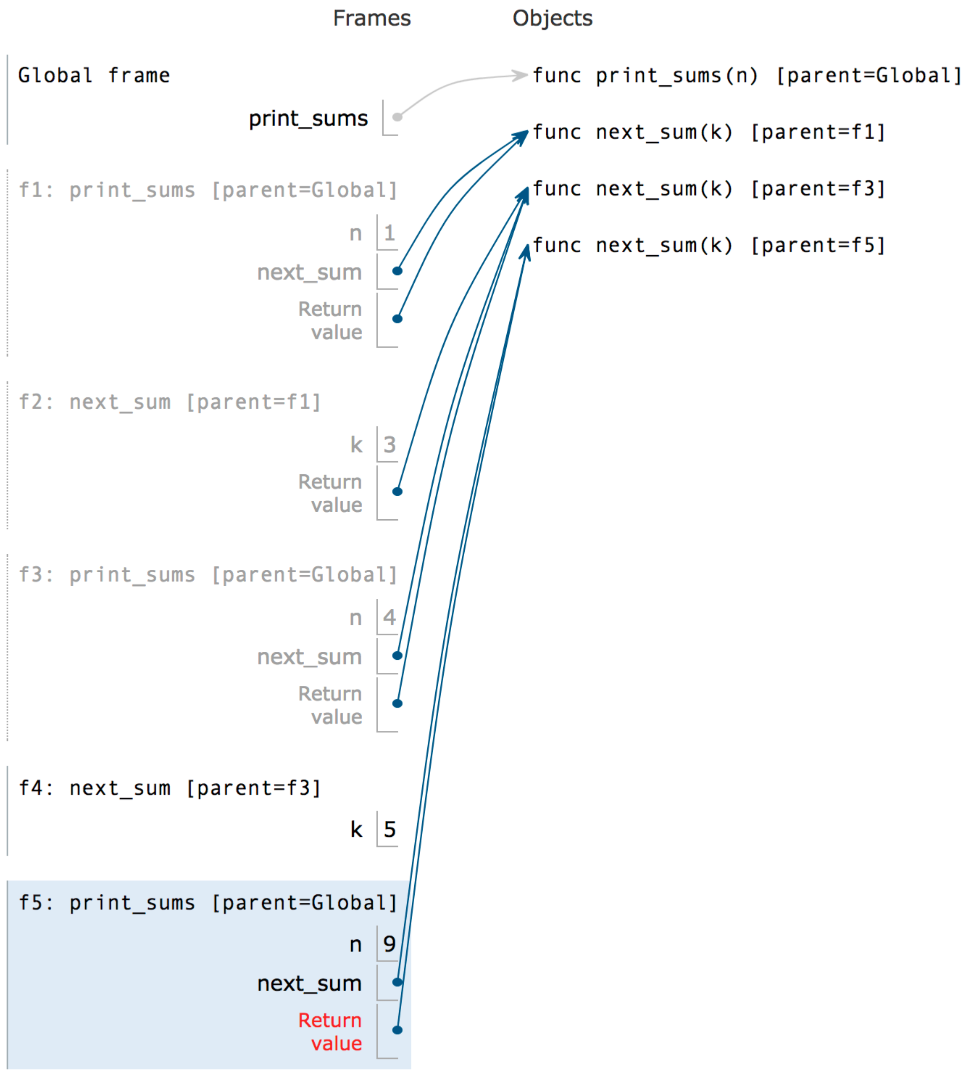
Self-Reference

`(Demo)`

Returning a Function Using Its Own Name

```
def print_sums(n):
    print(n)
    def next_sum(k):
        return print_sums(n+k)
    return next_sum

print_sums(1)(3)(5)
```



Frames
Objects
Global frame
print_sums
func print_sums(n) [parent=Global]
func next_sum(k) [parent=f1]
func next_sum(k) [parent=f3]
func next_sum(k) [parent=f5]
f1: print_sums [parent=Global]
n 1
next_sum
Return value
f2: next_sum [parent=f1]
k 3
Return value
f3: print_sums [parent=Global]
n 4
next_sum
Return value
f4: next_sum [parent=f3]
k 5
f5: print_sums [parent=Global]
n 9
next_sum
Return value

http://pythontutor.com/composingprograms.html#code=def%20print_all%28k%29%3A%0A%20%20%20%20print%28k%29%0A%20%20%20%20return%20print_all%0A%20%20%20%20%0Aprint_all%281%29%283%29%285%29&cumulative=true&curInstr=0&mode=display&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D

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Currying

Function Currying

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    return lambda k: n + k
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def make_adder(n):
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```
>>> make_adder(2)(3)
5
>>> add(2, 3)
5
```

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There's a general relationship between these functions

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(Demo)

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There's a general relationship between these functions

(Demo)

Curry: Transform a multi-argument function into a single-argument, higher-order function